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# NumPy

*Third Edition*

Build efficient, high-speed programs using the high-performance NumPy mathematical library

## *Beginner's Guide*

Ivan Idris

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BIRMINGHAM - MUMBAI

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## ***Third Edition***

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# Credits

**Author**

Ivan Idris

**Project Coordinator**

Shweta H. Birwatkar

**Reviewers**

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Davide Fiacconi

Ardo Illaste

**Proofreader**

Safis Editing

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Vikrant Phadke

Sameen Siddiqui

# About the Author

**Ivan Idris** has an MSc in experimental physics. His graduation thesis had a strong emphasis on applied computer science. After graduating, he worked for several companies as a Java developer, data warehouse developer, and QA Analyst. His main professional interests are business intelligence, big data, and cloud computing. Ivan enjoys writing clean, testable code and interesting technical articles. He is the author of *NumPy Beginner's Guide*, *NumPy Cookbook*, *Learning NumPy Array*, and *Python Data Analysis*. You can find more information about him and a blog with a few examples of NumPy at <http://ivanidris.net/wordpress/>.

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I would like to take this opportunity to thank the reviewers and the team at Packt Publishing for making this book possible. Also thanks go to my teachers, professors, colleagues, Wikipedia contributors, Stack Overflow contributors, and other authors who taught me science and programming. Last but not least, I would like to acknowledge my parents, family, and friends for their support.

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# About the Reviewers

**Davide Fiacconi** is completing his PhD in theoretical astrophysics from the Institute for Computational Science at the University of Zurich. He did his undergraduate and graduate studies at the University of Milan-Bicocca, studying the evolution of collisional ring galaxies using hydrodynamic numerical simulations. Davide's research now focuses on the formation and coevolution of supermassive black holes and galaxies, using both massively parallel simulations and analytical techniques. In particular, his interests include the formation of the first supermassive black hole seeds, the dynamics of binary black holes, and the evolution of high-redshift galaxies.

**Ardo Illaste** is a data scientist. He wants to provide everyone with easy access to data for making major life and career decisions. He completed his PhD in computational biophysics, prior to fully delving into data mining and machine learning. Ardo has worked and studied in Estonia, the USA, and Switzerland.

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*I dedicate this book to my aunt Lies who recently passed away. Rest in peace.*



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# Preface

Scientists, engineers, and quantitative data analysts face many challenges nowadays. Data scientists want to be able to perform numerical analysis on large datasets with minimal programming effort. They also want to write readable, efficient, and fast code that is as close as possible to the mathematical language they are used to. A number of accepted solutions are available in the scientific computing world.

The C, C++, and Fortran programming languages have their benefits, but they are not interactive and considered too complex by many. The common commercial alternatives, such as MATLAB, Maple, and Mathematica, provide powerful scripting languages that are even more limited than any general-purpose programming language. Other open source tools similar to MATLAB exist, such as R, GNU Octave, and Scilab. Obviously, they too lack the power of a language such as Python.

Python is a popular general-purpose programming language that is widely used in the scientific community. You can access legacy C, Fortran, or R code easily from Python. It is object-oriented and considered to be of a higher level than C or Fortran. It allows you to write readable and clean code with minimal fuss. However, it lacks an out-of-the-box MATLAB equivalent. That's where NumPy comes in. This book is about NumPy and related Python libraries, such as SciPy and matplotlib.

## What is NumPy?

NumPy (short for numerical Python) is an open source Python library for scientific computing. It lets you work with arrays and matrices in a natural way. The library contains a long list of useful mathematical functions, including some functions for linear algebra, Fourier transformation, and random number generation routines. LAPACK, a linear algebra library, is used by the NumPy linear algebra module if you have it installed on your system. Otherwise, NumPy provides its own implementation. LAPACK is a well-known library, originally written in Fortran, on which MATLAB relies as well. In a way, NumPy replaces some of the functionality of MATLAB and Mathematica, allowing rapid interactive prototyping.

We will not be discussing NumPy from a developing contributor's perspective, but from more of a user's perspective. NumPy is a very active project and has a lot of contributors. Maybe, one day you will be one of them!

## History

NumPy is based on its predecessor Numeric. Numeric was first released in 1995 and has deprecated status now. Neither Numeric nor NumPy made it into the standard Python library for various reasons. However, you can install NumPy separately, which will be explained in *Chapter 1, NumPy Quick Start*.

In 2001, a number of people inspired by Numeric created SciPy, an open source scientific computing Python library that provides functionality similar to that of MATLAB, Maple, and Mathematica. Around this time, people were growing increasingly unhappy with Numeric. Numarray was created as an alternative to Numeric. That is also deprecated now. It was better in some areas than Numeric, but worked very differently. For that reason, SciPy kept on depending on the Numeric philosophy and the Numeric array object. As is customary with new latest and greatest software, the arrival of Numarray led to the development of an entire ecosystem around it, with a range of useful tools.

In 2005, Travis Oliphant, an early contributor to SciPy, decided to do something about this situation. He tried to integrate some of Numarray's features into Numeric. A complete rewrite took place, and it culminated in the release of NumPy 1.0 in 2006. At that time, NumPy had all the features of Numeric and Numarray, and more. Tools were available to facilitate the upgrade from Numeric and Numarray. The upgrade is recommended since Numeric and Numarray are not actively supported any more.

Originally, the NumPy code was a part of SciPy. It was later separated and is now used by SciPy for array and matrix processing.

## Why use NumPy?

NumPy code is much cleaner than straight Python code and it tries to accomplish the same tasks. There are fewer loops required because operations work directly on arrays and matrices. The many convenience and mathematical functions make life easier as well. The underlying algorithms have stood the test of time and have been designed with high performance in mind.

NumPy's arrays are stored more efficiently than an equivalent data structure in base Python, such as a list of lists. Array IO is significantly faster too. The improvement in performance scales with the number of elements of the array. For large arrays, it really pays off to use NumPy. Files as large as several terabytes can be memory-mapped to arrays, leading to optimal reading and writing of data.

The drawback of NumPy arrays is that they are more specialized than plain lists. Outside the context of numerical computations, NumPy arrays are less useful. The technical details of NumPy arrays will be discussed in later chapters.

Large portions of NumPy are written in C. This makes NumPy faster than pure Python code. A NumPy C API exists as well, and it allows further extension of functionality with the help of the C language. The C API falls outside the scope of the book. Finally, since NumPy is open source, you get all the related advantages. The price is the lowest possible—as free as a beer. You don't have to worry about licenses every time somebody joins your team or you need an upgrade of the software. The source code is available for everyone. This of course is beneficial to code quality.

## Limitations of NumPy

If you are a Java programmer, you might be interested in Jython, the Java implementation of Python. In that case, I have bad news for you. Unfortunately, Jython runs on the Java Virtual Machine and cannot access NumPy because NumPy's modules are mostly written in C. You could say that Jython and Python are two totally different worlds, though they do implement the same specifications. There are some workarounds for this discussed in *NumPy Cookbook - Second Edition*, Packt Publishing, written by Ivan Idris.

## What this book covers

*Chapter 1, NumPy Quick Start*, guides you through the steps needed to install NumPy on your system and create a basic NumPy application.

*Chapter 2, Beginning with NumPy Fundamentals*, introduces NumPy arrays and fundamentals.

*Chapter 3, Getting Familiar with Commonly Used Functions*, teaches you the most commonly used NumPy functions—the basic mathematical and statistical functions.

*Chapter 4, Convenience Functions for Your Convenience*, tells you about functions that make working with NumPy easier. This includes functions that select certain parts of your arrays, for instance, based on a Boolean condition. You also learn about polynomials and manipulating the shapes of NumPy objects.

*Chapter 5, Working with Matrices and ufuncs*, covers matrices and universal functions. Matrices are well-known in mathematics and have their representation in NumPy as well. Universal functions (ufuncs) work on arrays element by element, or on scalars. ufuncs expect a set of scalars as the input and produce a set of scalars as the output.

*Chapter 6, Moving Further with NumPy Modules*, discusses a number of basic modules of universal functions. These functions can typically be mapped to their mathematical counterparts, such as addition, subtraction, division, and multiplication.

*Chapter 7, Peeking into Special Routines*, describes some of the more specialized NumPy functions. As NumPy users, we sometimes find ourselves having special requirements. Fortunately, NumPy satisfies most of our needs.

*Chapter 8, Assuring Quality with Testing*, teaches you how to write NumPy unit tests.

*Chapter 9, Plotting with matplotlib*, covers matplotlib in depth, a very useful Python plotting library. NumPy cannot be used on its own to create graphs and plots. matplotlib integrates nicely with NumPy and has plotting capabilities comparable to MATLAB.

*Chapter 10, When NumPy Is Not Enough – SciPy and Beyond*, covers more details about SciPy. We know that SciPy and NumPy are historically related. SciPy, as mentioned in the *History* section, is a high-level Python scientific computing framework built on top of NumPy. It can be used in conjunction with NumPy.

*Chapter 11, Playing with Pygame*, is the dessert of this book. You learn how to create fun games with NumPy and Pygame. You also get a taste of artificial intelligence in this chapter.

*Appendix A, Pop Quiz Answers*, has the answers to all the pop quiz questions within the chapters.

*Appendix B, Additional Online Resources*, contains links to Python, mathematics, and statistics websites.

*Appendix C, NumPy Functions' References*, lists some useful NumPy functions and their descriptions.

## What you need for this book

To try out the code samples in this book, you will need a recent build of NumPy. This means that you will need one of the Python versions supported by NumPy as well. Some code samples make use of matplotlib for illustration purposes. matplotlib is not strictly required to follow the examples, but it is recommended that you install it too. The last chapter is about SciPy and has one example involving SciKits.

Here is a list of the software used to develop and test the code examples:

- ◆ Python 2.7
- ◆ NumPy 1.9
- ◆ SciPy 0.13

- ◆ matplotlib 1.3.1
- ◆ Pygame 1.9.1
- ◆ IPython 2.4.1

Needless to say, you don't need exactly this software and these versions on your computer. Python and NumPy constitute the absolute minimum you will need.

## Who this book is for

This book is for the scientists, engineers, programmers, or analysts looking for a high-quality, open source mathematical library. Knowledge of Python is assumed. Also, some affinity, or at least interest, in mathematics and statistics is required. However, I have provided brief explanations and pointers to learning resources.

## Sections

In this book, you will find several headings that appear frequently (Time for action, What just happened?, Have a go hero, and Pop quiz).

To give clear instructions on how to complete a procedure or task, we use the following sections.

### **Time for action – heading**

- 1.** Action 1
- 2.** Action 2
- 3.** Action 3

Instructions often need some extra explanation to ensure that they make sense, so they are followed by these sections.

### ***What just happened?***

This section explains the working of the tasks or instructions that you have just completed.

You will also find some other learning aids in the book.



## Pop quiz – heading

These are short multiple-choice questions intended to help you test your own understanding.

## Have a go hero – heading

These are practical challenges that give you ideas to experiment with what you have learned.

## Conventions

In this book, you will find a number of styles of text that distinguish between different kinds of information. Here are some examples of these styles, and an explanation of their meaning.

Code words in text are shown as follows: "Notice that `numpysum()` does not need a `for` loop."

A block of code is set as follows:

```
def numpysum(n):
    a = numpy.arange(n) ** 2
    b = numpy.arange(n) ** 3
    c = a + b
    return c
```



When we wish to draw your attention to a particular part of a code block, the relevant lines or items are set in bold:



```
reals = np.isreal(xpoints)
print "Real number?", reals
Real number? [ True True True True False False False False]
```

Any command-line input or output is written as follows:

```
>>>fromnumpy.testing import rundocs
>>>rundocs('docstringtest.py')
```

**New terms** and **important words** are shown in bold. Words that you see on the screen, in menus or dialog boxes for example, appear in the text like this: "Clicking on the **Next** button moves you to the next screen."

 Warnings or important notes appear in a box like this. 

 Tips and tricks appear like this. 

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# 1

## NumPy Quick Start

*Let's get started. We will install NumPy and related software on different operating systems and have a look at some simple code that uses NumPy. This chapter briefly introduces the IPython interactive shell. SciPy is closely related to NumPy, so you will see the SciPy name appearing here and there. At the end of this chapter, you will find pointers on how to find additional information online if you get stuck or are uncertain about the best way to solve problems.*

In this chapter, you will cover the following topics:

- ◆ Install Python, SciPy, matplotlib, IPython, and NumPy on Windows, Linux, and Macintosh
- ◆ Do a short refresher of Python
- ◆ Write simple NumPy code
- ◆ Get to know IPython
- ◆ Browse online documentation and resources

### Python

**NumPy** is based on Python, so you need to have Python installed. On some operating systems, Python is already installed. However, you need to check whether the Python version corresponds with the NumPy version you want to install. There are many implementations of Python, including commercial implementations and distributions. In this book, we focus on the standard **CPython** implementation, which is guaranteed to be compatible with NumPy.

## Time for action – installing Python on different operating systems

NumPy has binary installers for Windows, various Linux distributions, and Mac OS X at <http://sourceforge.net/projects/numpy/files/>. There is also a source distribution, if you prefer that. You need to have Python 2.4.x or above installed on your system. We will go through the various steps required to install Python on the following operating systems:

- ◆ **Debian and Ubuntu:** Python might already be installed on Debian and Ubuntu, but the development headers are usually not. On Debian and Ubuntu, install the `python` and `python-dev` packages with the following commands:  

```
$ [sudo] apt-get install python  
$ [sudo] apt-get install python-dev
```
- ◆ **Windows:** The Windows Python installer is available at <https://www.python.org/downloads/>. On this website, we can also find installers for Mac OS X and source archives for Linux, UNIX, and Mac OS X.
- ◆ **Mac:** Python comes preinstalled on Mac OS X. We can also get Python through MacPorts, Fink, Homebrew, or similar projects.

Install, for instance, the Python 2.7 port by running the following command:

```
$ [sudo] port install python27
```

**Linear Algebra PACKage (LAPACK)** does not need to be present but, if it is, NumPy will detect it and use it during the installation phase. It is recommended that you install LAPACK for serious numerical analysis as it has useful numerical linear algebra functionality.

### What just happened?

We installed Python on Debian, Ubuntu, Windows, and the Mac OS X.



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## The Python help system

Before we start the NumPy introduction, let's take a brief tour of the Python help system, in case you have forgotten how it works or are not very familiar with it. The Python help system allows you to look up documentation from the interactive **Python shell**. A shell is an interactive program, which accepts commands and executes them for you.

### Time for action – using the Python help system

Depending on your operating system, you can access the Python shell with special applications, usually a terminal of some sort.

1. In such a terminal, type the following command to start a Python shell:  
`$ python`
2. You will get a short message with the Python version and other information and the following prompt:

```
>>>
```

Type the following in the prompt:

```
>>> help()
```

Another message appears and the prompt changes as follows:

```
help>
```

3. If you type, for instance, `keywords` as the message says, you get a list of keywords. The `topics` command gives a list of topics. If you type any of the topic names (such as `LISTS`) in the prompt, you get additional information about the topic. Typing `q` quits the information screen. Pressing `Ctrl + D` together returns you to the normal Python prompt:

```
>>>
```

Pressing `Ctrl + D` together again ends the Python shell session.

### What just happened?

We learned about the Python interactive shell and the Python help system.

## Basic arithmetic and variable assignment

In the *Time for action – using the Python help system* section, we used the Python shell to look up documentation. We can also use Python as a calculator. By the way, this is just a refresher, so if you are completely new to Python, I recommend taking some time to learn the basics. If you put your mind to it, learning basic Python should not take you more than a couple of weeks.

### Time for action – using Python as a calculator

We can use Python as a calculator as follows:

1. In a Python shell, add 2 and 2 as follows:

```
>>> 2 + 2
4
```

2. Multiply 2 and 2 as follows:

```
>>> 2 * 2
4
```

3. Divide 2 and 2 as follows:

```
>>> 2/2
1
```

4. If you have programmed before, you probably know that dividing is a bit tricky since there are different types of dividing. For a calculator, the result is usually adequate, but the following division may not be what you were expecting:

```
>>> 3/2
1
```

We will discuss what this result is about in several later chapters of this book. Take the cube of 2 as follows:

```
>>> 2 ** 3
8
```

### What just happened?

We used the Python shell as a calculator and performed addition, multiplication, division, and exponentiation.

## Time for action – assigning values to variables

Assigning values to variables in Python works in a similar way to most programming languages.

1. For instance, assign the value of 2 to a variable named `var` as follows:

```
>>> var = 2
>>> var
2
```

2. We defined the variable and assigned it a value. In this Python code, the type of the variable is not fixed. We can make the variable in to a list, which is a built-in Python type corresponding to an ordered sequence of values. Assign a list to `var` as follows:

```
>>> var = [2, 'spam', 'eggs']
>>> var
[2, 'spam', 'eggs']
```

We can assign a new value to a list item using its index number (counting starts from 0). Assign a new value to the first list element:

```
>>> var
['ham', 'spam', 'eggs']
```

3. We can also swap values easily. Define two variables and swap their values:

```
>>> a = 1
>>> b = 2
>>> a, b = b, a
>>> a
2
>>> b
1
```

### ***What just happened?***

We assigned values to variables and Python list items. This section is by no means exhaustive; therefore, if you are struggling, please read *Appendix B, Additional Online Resources*, to find recommended Python tutorials.



## The print() function

If you haven't programmed in Python for a while or are a Python novice, you may be confused about the Python 2 versus Python 3 discussions. In a nutshell, the latest version Python 3 is not backward compatible with the older Python 2 because the Python development team felt that some issues were fundamental and therefore warranted a radical change. The Python team has committed to maintain Python 2 until 2020. This may be problematic for the people who still depend on Python 2 in some way. The consequence for the `print()` function is that we have two types of syntax.

### Time for action – printing with the print() function

We can print using the `print()` function as follows:

1. The old syntax is as follows:

```
>>> print 'Hello'
Hello
```

2. The new Python 3 syntax is as follows:

```
>>> print('Hello')
Hello
```

The parentheses are now mandatory in Python 3. In this book, I try to use the new syntax as much as possible; however, I use Python 2 to be on the safe side. To enforce the syntax, each Python 2 script with `print()` calls in this book starts with:

```
>>> from __future__ import print_function
```

3. Try to use the old syntax to get the following error message:

```
>>> print 'Hello'
      File "<stdin>", line 1
        print 'Hello'
            ^
SyntaxError: invalid syntax
```

4. To print a newline, use the following syntax:

```
>>> print()
```

5. To print multiple items, separate them with commas:

```
>>> print(2, 'ham', 'egg')
2 ham egg
```

- By default, Python separates the printed values with spaces and prints output to the screen. You can customize these settings. Read more about this function by typing the following command:

```
>>> help(print)
```

You can exit again by typing `q`.

## What just happened?

We learned about the `print()` function and its relation to Python 2 and Python 3.

## Code comments

Commenting code is a best practice with the goal of making code clearer for yourself and other coders (see <https://google-styleguide.googlecode.com/svn/trunk/pyguide.html?showone=Comments#Comments>). Usually, companies and other organizations have policies regarding code comment such as comment templates. In this book, I did not comment the code in such a fashion for brevity and because the text in the book should clarify the code.

## Time for action – commenting code

The most basic comment starts with a hash sign and continues until the end of the line:

- Comment code with this type of comment as follows:

```
>>> # Comment from hash to end of line
```

- However, if the hash sign is between single or double quotes, then we have a string, which is an ordered sequence of characters:

```
>>> astring = '# This is not a comment'
>>> astring
'# This is not a comment'
```

- We can also comment multiple lines as a block. This is useful if you want to write a more detailed description of the code. Comment multiple lines as follows:

```
"""
    Chapter 1 of NumPy Beginners Guide.
    Another line of comment.
"""
```

We refer to this type of comment as triple-quoted for obvious reasons. It also is used to test code. You can read about testing in *Chapter 8, Assuring Quality with Testing*.

## The if statement

The `if` statement in Python has a bit different syntax to other languages, such as C++ and Java. The most important difference is that indentation matters, which I hope you are aware of.

### Time for action – deciding with the if statement

We can use the `if` statement in the following ways:

1. Check whether a number is negative as follows:

```
>>> if 42 < 0:
...     print('Negative')
... else:
...     print('Not negative')
...
Not negative
```

In the preceding example, Python decided that 42 is not negative. The `else` clause is optional. The comparison operators are equivalent to the ones in C++, Java, and similar languages.

2. Python also has a chained branching logic compound statement for multiple tests similar to the `switch` statement in C++, Java, and other programming languages. Decide whether a number is negative, 0, or positive as follows:

```
>>> a = -42
>>> if a < 0:
...     print('Negative')
... elif a == 0:
...     print('Zero')
... else:
...     print('Positive')
...
Negative
```

This time, Python decided that 42 is negative.

### What just happened?

We learned how to do branching logic in Python.

## The for loop

Python has a `for` statement with the same purpose as the equivalent construct in C++, Pascal, Java, and other languages. However, the mechanism of looping is a bit different.

### Time for action – repeating instructions with loops

We can use the `for` loop in the following ways:

1. Loop over an ordered sequence, such as a list, and print each item as follows:

```
>>> food = ['ham', 'egg', 'spam']
>>> for snack in food:
...     print(snack)
...
ham
egg
spam
```

2. And remember that, as always, indentation matters in Python. We loop over a range of values with the built-in `range()` or `xrange()` functions. The latter function is slightly more efficient in certain cases. Loop over the numbers 1-9 with a step of 2 as follows:

```
>>> for i in range(1, 9, 2):
...     print(i)
...
1
3
5
7
```

3. The start and step parameter of the `range()` function are optional with default values of 1. We can also prematurely end a loop. Loop over the numbers 0-9 and break out of the loop when you reach 3:

```
>>> for i in range(9):
...     print(i)
...     if i == 3:
...         print('Three')
...         break
```

```
...
0
1
2
3
Three
```

4. The loop stopped at 3 and we did not print the higher numbers. Instead of leaving the loop, we can also get out of the current iteration. Print the numbers 0-4, skipping 3 as follows:

```
>>> for i in range(5):
...     if i == 3:
...         print('Three')
...         continue
...     print(i)
...
0
1
2
Three
4
```

5. The last line in the loop was not executed when we reached 3 because of the `continue` statement. In Python, the `for` loop can have an `else` statement attached to it. Add an `else` clause as follows:

```
>>> for i in range(5):
...     print(i)
... else:
...     print(i, 'in else clause')
...
0
1
2
3
4
(4, 'in else clause')
```

6. Python executes the code in the `else` clause last. Python also has a `while` loop. I do not use it that much because the `for` loop is more useful in my opinion.

## What just happened?

We learned how to repeat instructions in Python with loops. This section included the `break` and `continue` statements, which exit and continue looping.

## Python functions

Functions are callable blocks of code. We call functions by the name we give them.

### Time for action – defining functions

Let's define the following simple function:

1. Print `Hello` and a given name in the following way:

```
>>> def print_hello(name):
...     print('Hello ' + name)
... 
```

Call the function as follows:

```
>>> print_hello('Ivan')
Hello Ivan
```

2. Some functions do not have arguments, or the arguments have default values. Give the function a default argument value as follows:

```
>>> def print_hello(name='Ivan'):
...     print('Hello ' + name)
...
>>> print_hello()
Hello Ivan
```

3. Usually, we want to return a value. Define a function, which doubles input values as follows:

```
>>> def double(number):
...     return 2 * number
...
>>> double(3)
6
```

## What just happened?

We learned how to define functions. Functions can have default argument values and return values.

## Python modules

A file containing Python code is called a **module**. A module can import other modules, functions in other modules, and other parts of modules. The filenames of Python modules end with `.py`. The name of the module is the same as the filename minus the `.py` suffix.

### Time for action – importing modules

Importing modules can be done in the following manner:

1. If the filename is, for instance, `mymodule.py`, import it as follows:

```
>>> import mymodule
```

2. The standard Python distribution has a `math` module. After importing it, list the functions and attributes in the module as follows:

```
>>> import math
```

```
>>> dir(math)
```

```
['__doc__', '__file__', '__name__', '__package__', 'acos',  
'acosh', 'asin', 'asinh', 'atan', 'atan2', 'atanh', 'ceil',  
'copysign', 'cos', 'cosh', 'degrees', 'e', 'erf', 'erfc', 'exp',  
'expm1', 'fabs', 'factorial', 'floor', 'fmod', 'frexp', 'fsum',  
'gamma', 'hypot', 'isinf', 'isnan', 'ldexp', 'lgamma', 'log',  
'log10', 'log1p', 'modf', 'pi', 'pow', 'radians', 'sin', 'sinh',  
'sqrt', 'tan', 'tanh', 'trunc']
```

3. Call the `pow()` function in the `math` module:

```
>>> math.pow(2, 3)
```

```
8.0
```

Notice the dot in the syntax. We can also import a function directly and call it by its short name. Import and call the `pow()` function as follows:

```
>>> from math import pow
```

```
>>> pow(2, 3)
```

```
8.0
```

4. Python lets us define aliases for imported modules and functions. This is a good time to introduce the import conventions we are going to use for NumPy and a plotting library we will use a lot:

```
import numpy as np
import matplotlib.pyplot as plt
```

### ***What just happened?***

We learned about modules, importing modules, importing functions, calling functions in modules, and the import conventions of this book. This concludes the Python refresher.

## **NumPy on Windows**

Installing NumPy on Windows is straightforward. You only need to download an installer, and a wizard will guide you through the installation steps.

### **Time for action – installing NumPy, matplotlib, SciPy, and IPython on Windows**

Installing NumPy on Windows is necessary but this is, fortunately, a straightforward task that we will cover in detail. It is recommended that you install matplotlib, SciPy, and IPython. However, this is not required to enjoy this book. The actions we will take are as follows:

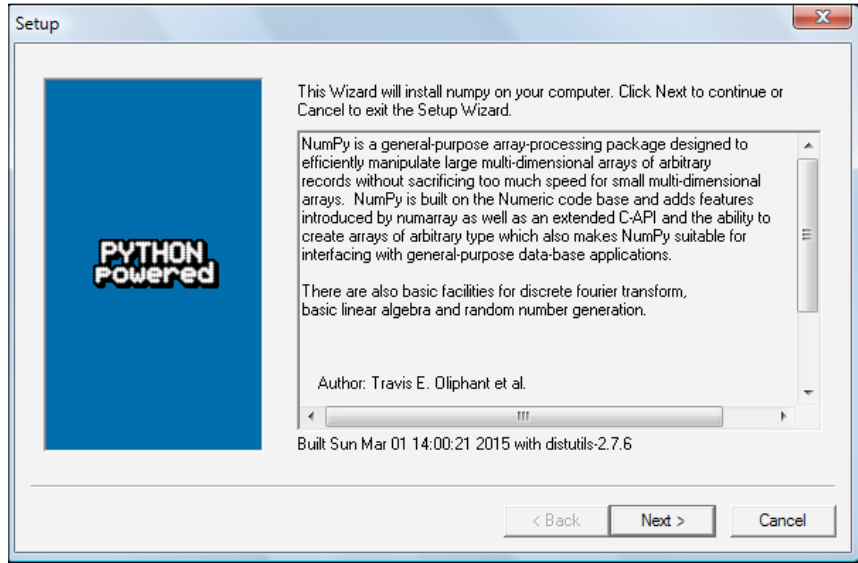
1. Download a NumPy installer for Windows from the **SourceForge** website <http://sourceforge.net/projects/numpy/files/>.



Choose the appropriate NumPy version according to your Python version. In the preceding screen shot, we chose `numpy-1.9.2-win32-superpack-python2.7.exe`.



2. Open the EXE installer by double-clicking on it as shown in the following screen shot:



3. Now, we can see a description of NumPy and its features. Click on **Next**.
4. If you have Python installed, it should automatically be detected. If it is not detected, your path settings might be wrong. At the end of this chapter, we have listed resources in case you have problems with installing NumPy.
5. In this example, Python 2.7 was found. Click on **Next** if Python is found; otherwise, click on **Cancel** and install Python (NumPy cannot be installed without Python). Click on **Next**. This is the point of no return. Well, kind of, but it is best to make sure that you are installing to the proper directory and so on and so forth. Now the real installation starts. This may take a while.

Install SciPy and matplotlib with the **Enthought Canopy** distribution (<https://www.enthought.com/products/canopy/>). It might be necessary to put the `msvcp71.dll` file in your `C:\Windows\system32` directory. You can get it from <http://www.dll-files.com/dllindex/dll-files.shtml?msvcp71>  
A Windows IPython installer is available on the IPython website (see <http://ipython.org/>).

## ***What just happened?***

We installed NumPy, SciPy, matplotlib, and IPython on Windows.

## NumPy on Linux

Installing NumPy and its related recommended software on Linux depends on the distribution you have. We will discuss how you will install NumPy from the command line, although you can probably use graphical installers; it depends on your **distribution (distro)**. The commands to install matplotlib, SciPy, and IPython are the same—only the package names are different. Installing matplotlib, SciPy, and IPython is recommended, but optional.

### Time for action – installing NumPy, matplotlib, SciPy, and IPython on Linux

Most Linux distributions have NumPy packages. We will go through the necessary commands for some of the most popular Linux distros:

- ◆ **Installing NumPy on Red Hat:** Run the following instructions from the command line:  

```
$ yum install python-numpy
```
- ◆ **Installing NumPy on Mandriva:** To install NumPy on Mandriva, run the following command line instruction:  

```
$ urpmi python-numpy
```
- ◆ **Installing NumPy on Gentoo:** To install NumPy on Gentoo, run the following command line instruction:  

```
$ [sudo] emerge numpy
```
- ◆ **Installing NumPy on Debian and Ubuntu:** On Debian or Ubuntu, type the following on the command line:  

```
$ [sudo] apt-get install python-numpy
```

The following table gives an overview of the Linux distributions and the corresponding package names for NumPy, SciPy, matplotlib, and IPython:

Linux distribution	NumPy	SciPy	matplotlib	IPython
Arch Linux	python-numpy	python-scipy	python-matplotlib	ipython
Debian	python-numpy	python-scipy	python-matplotlib	ipython
Fedora	numpy	python-scipy	python-matplotlib	ipython
Gentoo	dev-python/numpy	scipy	matplotlib	ipython

Linux distribution	NumPy	SciPy	matplotlib	IPython
OpenSUSE	python-numpy, python-numpy- devel	python- scipy	python- matplotlib	ipython
Slackware	numpy	scipy	matplotlib	ipython

## NumPy on Mac OS X

You can install NumPy, matplotlib, and SciPy on the Mac OS X with a GUI installer (not possible for all versions) or from the command line with a port manager such as **MacPorts**, **Homebrew**, or **Fink**, depending on your preference. You can also install using a script from <https://github.com/fonnesbeck/ScipySuperpack>.

### Time for action – installing NumPy, SciPy, matplotlib, and IPython with MacPorts or Fink

Alternatively, we can install NumPy, SciPy, matplotlib, and IPython through the MacPorts route or with Fink. The following installation steps show how to install all these packages:

- ◆ **Installing with MacPorts:** Type the following command:  

```
$ [sudo] port install py-numpy py-scipy py-matplotlib py-ipython
```
- ◆ **Installing with Fink:** Fink also has packages for NumPy—`scipy-core-py24`, `scipy-core-py25`, and `scipy-core-py26`. The SciPy packages are `scipy-py24`, `scipy-py25` and `scipy-py26`. We can install NumPy and the additional recommended packages, referring to this book on Python 2.7, using the following command:  

```
$ fink install scipy-core-py27 scipy-py27 matplotlib-py27
```

### What just happened?

We installed NumPy and the additional recommended software on Mac OS X with MacPorts and Fink.

## Building from source

We can retrieve the source code for NumPy with `git` as follows:

```
$ git clone git://github.com/numpy/numpy.git numpy
```

Alternatively, download the source from <http://sourceforge.net/projects/numpy/files/>.

Install in `/usr/local` with the following command:

```
$ python setup.py build
$ [sudo] python setup.py install --prefix=/usr/local
```

To build, we need a C compiler such as GCC and the Python header files in the `python-dev` or `python-devel` packages.

## Arrays

After going through the installation of NumPy, it's time to have a look at NumPy arrays. NumPy arrays are more efficient than Python lists when it comes to numerical operations. NumPy code requires less explicit loops than the equivalent Python code.

### Time for action – adding vectors

Imagine that we want to add two vectors called `a` and `b` (see <https://www.khanacademy.org/science/physics/one-dimensional-motion/displacement-velocity-time/v/introduction-to-vectors-and-scalars>). **Vector** is used here in the mathematical sense meaning a one-dimensional array. We will learn in *Chapter 5, Working with Matrices and ufuncs*, about specialized NumPy arrays, which represent matrices. Vector `a` holds the squares of integers 0 to `n`, for instance, if `n` is equal to 3, then `a` is equal to (0, 1, 4). Vector `b` holds the cubes of integers 0 to `n`, so if `n` is equal to 3, then `b` is equal to (0, 1, 8). How will you do that using plain Python? After we come up with a solution, we will compare it to the NumPy equivalent.

- 1. Adding vectors using pure Python:** The following function solves the vector addition problem using pure Python without NumPy:

```
def pythonsum(n):
    a = range(n)
    b = range(n)
    c = []

    for i in range(len(a)):
        a[i] = i ** 2
        b[i] = i ** 3
        c.append(a[i] + b[i])

    return c
```

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**2. Adding vectors using NumPy:** Following is a function that achieves the same result with NumPy:

```
def numpysum(n):
    a = np.arange(n) ** 2
    b = np.arange(n) ** 3
    c = a + b

    return c
```

Notice that `numpysum()` does not need a `for` loop. Also, we used the `arange()` function from NumPy that creates a NumPy array for us with integers 0 to  $n$ . The `arange()` function was imported; that is why it is prefixed with `numpy` (actually, it is customary to abbreviate it via an alias to `np`).

Now comes the fun part. The preface mentions that NumPy is faster when it comes to array operations. How much faster is NumPy, though? The following program will show us by measuring the elapsed time, in microseconds, for the `numpysum()` and `pythonsum()` functions. It also prints the last two elements of the vector sum. Let's check that we get the same answers by using Python and NumPy:

```
#!/usr/bin/env/python

from __future__ import print_function
import sys
from datetime import datetime
import numpy as np

"""
Chapter 1 of NumPy Beginners Guide.
This program demonstrates vector addition the Python way.
Run from the command line as follows

python vectorsum.py n

where n is an integer that specifies the size of the vectors.
```

---

The first vector to be added contains the squares of 0 up to n.  
The second vector contains the cubes of 0 up to n.  
The program prints the last 2 elements of the sum and the elapsed time.

```
"""
```

```
def numpysum(n):  
    a = np.arange(n) ** 2  
    b = np.arange(n) ** 3  
    c = a + b  
  
    return c  
  
def pythonsum(n):  
    a = range(n)  
    b = range(n)  
    c = []  
  
    for i in range(len(a)):  
        a[i] = i ** 2  
        b[i] = i ** 3  
        c.append(a[i] + b[i])  
  
    return c
```

```
size = int(sys.argv[1])
```

```
start = datetime.now()  
c = pythonsum(size)  
delta = datetime.now() - start  
print("The last 2 elements of the sum", c[-2:])  
print("PythonSum elapsed time in microseconds", delta.microseconds)
```

```
start = datetime.now()  
c = numpysum(size)  
delta = datetime.now() - start  
print("The last 2 elements of the sum", c[-2:])  
print("NumPySum elapsed time in microseconds", delta.microseconds)
```

The output of the program for 1000, 2000, and 3000 vector elements is as follows:

```
$ python vectorsum.py 1000
The last 2 elements of the sum [995007996, 998001000]
PythonSum elapsed time in microseconds 707
The last 2 elements of the sum [995007996 998001000]
NumPySum elapsed time in microseconds 171
$ python vectorsum.py 2000
The last 2 elements of the sum [7980015996, 7992002000]
PythonSum elapsed time in microseconds 1420
The last 2 elements of the sum [7980015996 7992002000]
NumPySum elapsed time in microseconds 168
$ python vectorsum.py 4000
The last 2 elements of the sum [63920031996, 63968004000]
PythonSum elapsed time in microseconds 2829
The last 2 elements of the sum [63920031996 63968004000]
NumPySum elapsed time in microseconds 274
```

### ***What just happened?***

Clearly, NumPy is much faster than the equivalent normal Python code. One thing is certain, we get the same results whether we use NumPy or not. However, the result printed differs in representation. Notice that the result from the `numpysum()` function does not have any commas. How come? Obviously, we are not dealing with a Python list but with a NumPy array. It was mentioned in the *Preface* that NumPy arrays are specialized data structures for numerical data. We will learn more about NumPy arrays in the next chapter.

### **Pop quiz – Functioning of the `arange()` function**

Q1. What does `arange(5)` do?

1. Creates a Python list of 5 elements with the values 1-5.
2. Creates a Python list of 5 elements with the values 0-4.
3. Creates a NumPy array with the values 1-5.
4. Creates a NumPy array with the values 0-4.
5. None of the above.

## Have a go hero – continue the analysis

The program we used to compare the speed of NumPy and regular Python is not very scientific. We should at least repeat each measurement a couple of times. It will be nice to be able to calculate some statistics such as average times. Also, you might want to show plots of the measurements to friends and colleagues.



Hints to help can be found in the online documentation and the resources listed at the end of this chapter. NumPy has statistical functions that can calculate averages for you. I recommend using matplotlib to produce plots. *Chapter 9, Plotting with matplotlib*, gives a quick overview of matplotlib.

## IPython – an interactive shell

Scientists and engineers are used to experiment. Scientists created **IPython** with experimentation in mind. Many view the interactive environment that IPython provides as a direct answer to **MATLAB**, **Mathematica**, and **Maple**. You can find more information, including installation instructions, at <http://ipython.org/>.

IPython is free, open source, and available for Linux, UNIX, Mac OS X, and Windows. The IPython authors only request that you cite IPython in any scientific work that uses IPython. The following is a list of the basic IPython features:

- ◆ Tab completion
- ◆ History mechanism
- ◆ Inline editing
- ◆ Ability to call external Python scripts with %run
- ◆ Access to system commands
- ◆ Pylab switch
- ◆ Access to Python debugger and profiler

The Pylab switch imports all the SciPy, NumPy, and matplotlib packages. Without this switch, we will have to import every package we need ourselves.



All we need to do is enter the following instruction on the command line:

```
$ ipython --pylab
IPython 2.4.1 -- An enhanced Interactive Python.
?          -> Introduction and overview of IPython's features.
%quickref  -> Quick reference.
help       -> Python's own help system.
object?    -> Details about 'object', use 'object??' for extra details.
Using matplotlib backend: MacOSX
```

```
In [1]: quit()
```

The `quit()` command or *Ctrl + D* quits the IPython shell. We may want to be able to go back to our experiments. In IPython, it is easy to save a session for later:

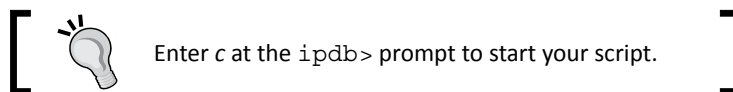
```
In [1]: %logstart
Activating auto-logging. Current session state plus future input saved.
Filename      : ipython_log.py
Mode          : rotate
Output logging : False
Raw input log  : False
Timestamping  : False
State         : active
```

Let's say we have the vector addition program that we made in the current directory. Run the script as follows:

```
In [1]: ls
README      vectorsum.py
In [2]: %run -i vectorsum.py 1000
```

As you probably remember, 1000 specifies the number of elements in a vector. The `-d` switch of `%run` starts an `ipdb` debugger with `c` the script is started. `n` steps through the code. Typing `quit` at the `ipdb` prompt exits the debugger:

```
In [2]: %run -d vectorsum.py 1000
*** Blank or comment
*** Blank or comment
Breakpoint 1 at: /Users/.../vectorsum.py:3
```



Enter `c` at the `ipdb>` prompt to start your script.

```
><string>(1)<module>()
ipdb> c
> /Users/.../vectorsum.py(3)<module>()
      2
1----> 3 import sys
      4 from datetime import datetime
ipdb> n
>
/Users/.../vectorsum.py(4)<module>()
1      3 import sys
----> 4 from datetime import datetime
      5 import numpy
ipdb> n
> /Users/.../vectorsum.py(5)<module>()
      4 from datetime import datetime
----> 5 import numpy
      6
ipdb> quit
```

We can also profile our script by passing the `-p` option to `%run`:

```
In [4]: %run -p vectorsum.py 1000
          1058 function calls (1054 primitive calls) in 0.002 CPU seconds
          Ordered by: internal time
ncalls tottime pcall  cumtime pcall  filename:lineno(function)
1 0.001  0.001  0.001  0.001 vectorsum.py:28(pythonsum)
1 0.001  0.001  0.002  0.002 {execfile}
1000 0.000  0.0000.0000.000 {method 'append' of 'list' objects}
1 0.000  0.000  0.002  0.002 vectorsum.py:3(<module>)
1 0.000  0.0000.0000.000 vectorsum.py:21(numpysum)
3  0.000  0.0000.0000.000 {range}
1  0.000  0.0000.0000.000 arrayprint.py:175(_array2string)
3/1  0.000  0.0000.0000.000 arrayprint.py:246(array2string)
```

```
2    0.000    0.0000.0000.000 {method 'reduce' of 'numpy.ufunc' objects}
4    0.000    0.0000.0000.000 {built-in method now}
2    0.000    0.0000.0000.000 arrayprint.py:486(_formatInteger)
2    0.000    0.0000.0000.000 {numpy.core.multiarray.arange}
1    0.000    0.0000.0000.000 arrayprint.py:320(_formatArray)
3/1  0.000    0.0000.0000.000 numeric.py:1390(array_str)
1    0.000    0.0000.0000.000 numeric.py:216(asarray)
2    0.000    0.0000.0000.000 arrayprint.py:312(_extendLine)
1    0.000    0.0000.0000.000 fromnumeric.py:1043(ravel)
2    0.000    0.0000.0000.000 arrayprint.py:208(<lambda>)
1    0.000    0.000    0.002    0.002<string>:1(<module>)
11   0.000    0.0000.0000.000 {len}
2    0.000    0.0000.0000.000 {isinstance}
1    0.000    0.0000.0000.000 {reduce}
1    0.000    0.0000.0000.000 {method 'ravel' of 'numpy.ndarray' objects}
4    0.000    0.0000.0000.000 {method 'rstrip' of 'str' objects}
3    0.000    0.0000.0000.000 {issubclass}
2    0.000    0.0000.0000.000 {method 'item' of 'numpy.ndarray' objects}
1    0.000    0.0000.0000.000 {max}
1    0.000    0.0000.0000.000 {method 'disable' of '_lsprof.Profiler'
objects}
```

This gives us a bit more insight in to the workings of our program. In addition, we can now identify performance bottlenecks. The `%hist` command shows the commands history:

```
In [2]: a=2+2
In [3]: a
Out[3]: 4
In [4]: %hist
1: _ip.magic("hist ")
2: a=2+2
3: a
```

I hope you agree that IPython is a really useful tool!

## Online resources and help

When we are in IPython's `pylab` mode, we can open manual pages for NumPy functions with the `help` command. It is not necessary to know the name of a function. We can type a few characters and then let tab completion do its work. Let's, for instance, browse the available information for the `arange()` function:

```
In [2]: help ar<Tab>
```

arange	arctan	argsort	array_equal	arrow
arccos	arctan2	argwhere	array_equiv	
arccosh	arctanh	around	array_repr	
arcsin	argmax	array	array_split	
arcsinh	argmin	array2string	array_str	

```
In [2]: help arange
```

Another option is to put a question mark behind the function name:

```
In [3]: arange?
```

The main documentation website for NumPy and SciPy is at <http://docs.scipy.org/doc/>. Through this web page, we can browse the NumPy reference at <http://docs.scipy.org/doc/numpy/reference/>, the user guide, and several tutorials.

The popular Stack Overflow software development forum has hundreds of questions tagged `numpy`. To view them, go to <http://stackoverflow.com/questions/tagged/numpy>.

If you are really stuck with a problem or you want to be kept informed of NumPy development, you can subscribe to the NumPy discussion mailing list. The e-mail address is [numpy-discussion@scipy.org](mailto:numpy-discussion@scipy.org). The number of e-mails per day is not too high with almost no spam to speak of. Most importantly, the developers actively involved with NumPy also answer questions asked on the discussion group. The complete list can be found at <http://www.scipy.org/scipylib/mailling-lists.html>.

For IRC users, there is an IRC channel on <irc://irc.freenode.net>. The channel is called `#scipy`, but you can also ask NumPy questions since SciPy users also have knowledge of NumPy, as SciPy is based on NumPy. There are at least 50 members on the SciPy channel at all times.

## **Summary**

In this chapter, we installed NumPy and other recommended software that we will be using in some sections of this book. We got a vector addition program working and convinced ourselves that NumPy has superior performance. You were introduced to the IPython interactive shell. In addition, you explored the available NumPy documentation and online resources.

In the next chapter, you will take a look under the hood and explore some fundamental concepts including arrays and data types.

# 2

## Beginning with NumPy Fundamentals

*After installing NumPy and getting some code to work, it's time to cover NumPy basics.*

The topics we shall cover in this chapter are as follows:

- ◆ Data types
- ◆ Array types
- ◆ Type conversions
- ◆ Array creation
- ◆ Indexing
- ◆ Slicing
- ◆ Shape manipulation

Before we start, let me make a few remarks about the code examples in this chapter. The code snippets in this chapter show input and output from several IPython sessions. Recall that IPython was introduced in *Chapter 1, NumPy Quick Start*, as the interactive Python shell of choice for scientific computing. The advantages of IPython are the `--pylab` switch that imports many scientific computing Python packages, including NumPy, and the fact that it is not necessary to explicitly call the `print()` function to display variable values. Other features include easy parallel computation and the notebook interface in the form of a persistent worksheet in a web browser.

However, the source code delivered alongside the book is a regular Python code that uses `import` and `print` statements.

## NumPy array object

NumPy has a multidimensional array object called `ndarray`. It consists of two parts:

- ◆ The actual data
- ◆ Some metadata describing the data

The majority of array operations leave the raw data untouched. The only aspect that changes is the metadata.

In the previous chapter, we have already learned how to create an array using the `arange()` function. Actually, we created a one-dimensional array that contained a set of numbers. The `ndarray` object can have more than one dimension.

The NumPy array is in general homogeneous (there is a special array type that is heterogeneous as described in the *Time for action – creating a record data type* section)—the items in the array have to be of the same type. The advantage is that, if we know that the items in the array are of the same type, it is easy to determine the storage size required for the array.

NumPy arrays are indexed starting from 0, just like in Python. Data types are represented by special objects. We will discuss these objects comprehensively in this chapter.

Let's create an array with the `arange()` function again. Get the data type of an array using the following code:

```
In: a = arange(5)
In: a.dtype
Out: dtype('int64')
```

The data type of array `a` is `int64` (at least on my machine), but you may get `int32` as output if you are using 32-bit Python. In both the cases, we are dealing with integers (64-bit or 32-bit). Besides the data type of an array, it is important to know its shape.

In *Chapter 1, NumPy Quick Start*, we demonstrated how to create a vector (actually, a one-dimensional NumPy array). A vector is commonly used in mathematics, but most of the time, we need higher dimensional objects. Determine the shape of the vector we created a few minutes ago. The following code is an example of creating a vector:

```
In [4]: a
Out[4]: array([0, 1, 2, 3, 4])
In: a.shape
Out: (5,)
```

As you can see, the vector has five elements with values ranging from 0 to 4. The shape attribute of the array is a tuple, in this case a tuple of 1 element, which contains the length in each dimension.



A **tuple** in Python is an immutable (it can't change) sequence of values. Once tuples are created, we are not allowed to change the values of tuple elements or append new elements. This makes tuples safer than lists because you can't mutate them by accident. A common use case for tuples is as return value of functions. For more examples, have a look at the *Introducing Tuples* section of *Chapter 3, Dive into Python*, available at [http://www.diveintopython.net/native\\_data\\_types/tuples.html](http://www.diveintopython.net/native_data_types/tuples.html).

## Time for action – creating a multidimensional array

Now that we know how to create a vector, we are ready to create a multidimensional NumPy array. After we create the array, we will again want to display its shape:

1. Create a two-by-two array:

```
In: m = array([arange(2), arange(2)])
In: m
Out:
array([[0, 1],
       [0, 1]])
```

2. Show the array shape:

```
In: m.shape
Out: (2, 2)
```

### What just happened?

We created a two-by-two array with the `arange()` and `array()` functions we have come to trust and love. Without any warning, the `array()` function appeared on the stage.

The `array()` function creates an array from an object that you give to it. The object needs to be array-like, for instance, a Python list. In the preceding example, we passed in a list of arrays. The object is the only required argument of the `array()` function. NumPy functions tend to have a lot of optional arguments with predefined defaults. View the documentation for this function from the IPython shell with the `help()` function given here:

```
In [1]: help(array)
```

Or use the following shorthand:

```
In [2]: array?
```

Of course, you can substitute `array` in this example with another NumPy function you are interested in.



## Pop quiz – the shape of ndarray

Q1. How is the shape of an ndarray stored?

1. It is stored in a comma-separated string.
2. It is stored in a list.
3. It is stored in a tuple.

## Have a go hero – create a three-by-three array

It shouldn't be too hard now to create a three-by-three array. Give it a go and check whether the array shape is as expected.

## Selecting elements

From time to time, we will want to select a particular element of an array. We will take a look at how to do this, but, first, create a two-by-two array again:

```
In: a = array([[1,2],[3,4]])
In: a
Out:
array([[1, 2],
       [3, 4]])
```

The array was created this time by passing a list of lists to the `array()` function. We will now select one by one each item of the matrix. Remember, the indices are numbered starting from 0 :

```
In: a[0,0]
Out: 1
In: a[0,1]
Out: 2
In: a[1,0]
Out: 3
In: a[1,1]
Out: 4
```

As you can see, selecting elements of the array is pretty simple. For the array `a`, we just use the notation `a[m,n]`, where `m` and `n` are the indices of the item in the array (the array can have even more dimensions than in this example). This screenshot shows a simple example of an array:

[0,0]	[0,1]
[1,0]	[1,1]

## NumPy numerical types

Python has an integer type, a float type, and a complex type; however, this is not enough for scientific computing and, for this reason, NumPy has a lot more data types with varying precision, dependent on memory requirements.



Integers represent whole numbers, such as -1, 0, and 1. Floating-point numbers correspond to real numbers as used in mathematics, for example, fractions or irrational numbers such as  $\pi$ . Because of the way computers work, we are able to represent integers exactly, but floating-point numbers are approximated. Complex numbers can have an imaginary component usually denoted with  $i$  or  $j$ . By definition,  $i$  is the square root of -1. For instance,  $2.5 + 3.7i$  is a complex number (for more information, refer to [https://www.khanacademy.org/math/precalculus/imaginary\\_complex\\_precalc](https://www.khanacademy.org/math/precalculus/imaginary_complex_precalc)).

In practice, we need even more types with varying precision and, therefore, different memory size of the type. The majority of the NumPy numerical types end with a number. This number indicates the number of bits associated with the type. The following table (adapted from the NumPy user guide) gives an overview of NumPy numerical types:

Type	Description
bool	Boolean (True or False) stored as a bit
int1	Platform integer (normally either int32 or int64)
int8	Byte (-128 to 127)
int16	Integer (-32768 to 32767)
int32	Integer ( $-2^{31}$ to $2^{31} - 1$ )
int64	Integer ( $-2^{63}$ to $2^{63} - 1$ )
uint8	Unsigned integer (0 to 255)
uint16	Unsigned integer (0 to 65535)
uint32	Unsigned integer (0 to $2^{32} - 1$ )
uint64	Unsigned integer (0 to $2^{64} - 1$ )
float16	Half precision float: sign bit, 5 bits exponent, 10 bits mantissa
float32	Single precision float: sign bit, 8 bits exponent, 23 bits mantissa
float64 or float	Double precision float: sign bit, 11 bits exponent, 52 bits mantissa

Type	Description
complex64	Complex number, represented by two 32-bit floats (real and imaginary components)
complex128 or complex	Complex number, represented by two 64-bit floats (real and imaginary components)

For floating-point types, we can request information with the `finfo()` function given here:

```
In: finfo(float16)
Out: finfo(resolution=0.0010004, min=-6.55040e+04, max=6.55040e+04,
dtype=float16)
```

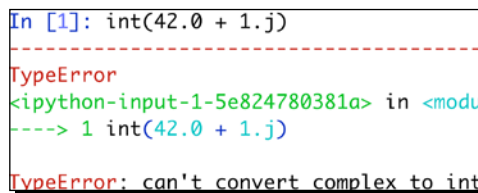
For each data type, there exists a corresponding conversion function:

```
In: float64(42)
Out: 42.0
In: int8(42.0)
Out: 42
In: bool(42)
Out: True
In: bool(0)
Out: False
In: bool(42.0)
Out: True
In: float(True)
Out: 1.0
In: float(False)
Out: 0.0
```

Many functions have a data type argument, which is often optional:

```
In: arange(7, dtype=uint16)
Out: array([0, 1, 2, 3, 4, 5, 6], dtype=uint16)
```

It is important to know that you are not allowed to convert a complex number into an integer or float. Trying to do that triggers a `TypeError`, as shown in the following screenshot:



```
In [1]: int(42.0 + 1.j)
-----
TypeError
<ipython-input-1-5e824780381a> in <module>
----> 1 int(42.0 + 1.j)
TypeError: can't convert complex to int
```

The same goes for conversion of a complex number into a float.



An exception in Python is an abnormal condition, which we usually try to avoid. A `TypeError` is a Python built-in exception, occurring when we specify the wrong type for an argument.

The `j` part is the imaginary coefficient of the complex number. However, you can convert a float in to a complex number, for instance, `complex(1.0)`.

## Data type objects

**Data type objects** are instances of the `numpy.dtype` class. Once again, arrays have a data type. To be precise, every element in a NumPy array has the same data type. The data type object can tell you the size of the data in bytes. The size in bytes is given by the `itemsize` attribute of the `dtype` class:

```
In: a.dtype.itemsize
```

```
Out: 8
```

## Character codes

**Character codes** are included for backward compatibility with Numeric. Numeric is the predecessor of NumPy. Their use is not recommended, but the codes are provided here because they pop up in several places. We should instead use the `dtype` objects. The table shows the character codes:

Type	Character code
Integer	i
Unsigned integer	u
Single precision float	f
Double precision float	d
Boolean	b
Complex	D
String	S
Unicode	U
Void	V

Look at the following code to create an array of single precision floats:

```
In: arange(7, dtype='f')
```

```
Out: array([ 0.,  1.,  2.,  3.,  4.,  5.,  6.], dtype=float32)
```

Likewise this creates an array of complex numbers.

```
In: arange(7, dtype='D')
Out: array([ 0.+0.j,  1.+0.j,  2.+0.j,  3.+0.j,  4.+0.j,  5.+0.j,
  6.+0.j])
```

## The dtype constructors

Python classes have functions, which are called **methods**, if they belong to a class. Some of these methods are special and used to create new objects. These specialized methods are called **constructors**.



You can read more about Python classes at <https://docs.python.org/2/tutorial/classes.html>.

We have a variety of ways to create data types. Take the case of floating point data:

- ◆ Use the general Python float:  
In: `dtype(float)`  
Out: `dtype('float64')`
- ◆ Specify a single precision float with a character code:  
In: `dtype('f')`  
Out: `dtype('float32')`
- ◆ Use a double precision float character code:  
In: `dtype('d')`  
Out: `dtype('float64')`
- ◆ We can give the data type constructor a two-character code. The first character signifies the type and the second character is a number specifying the number of bytes in the type (the numbers 2, 4, and 8 correspond to 16, 32, and 64-bit floats):  
In: `dtype('f8')`  
Out: `dtype('float64')`

A listing of all full data type names can be found with the `sctypeDict.keys()` function:

```
In: sctypeDict.keys()
Out: [0, ...
      'i2',
      'int0']
```

## The dtype attributes

The `dtype` class has a number of useful attributes. For example, get information about the character code of a data type through the attributes of `dtype`:

```
In: t = dtype('Float64')
In: t.char
Out: 'd'
```

The `type` attribute corresponds to the type of object of the array elements:

```
In: t.type
Out: <type 'numpy.float64'>
```

The `str` attribute of the `dtype` class gives a string representation of the data type. It starts with a character representing **endianness**, if appropriate, then a character code, followed by a number corresponding to the number of bytes that each array item requires. Endianness, here, refers to the way bytes are ordered within a 32- or 64-bit word. In big-endian order, the most significant byte is stored first, indicated by `>`. In little-endian order, the least significant byte is stored first, indicated by `<`:

```
In: t.str
Out: '<f8'
```

## Time for action – creating a record data type

The record data type is a heterogeneous data type—think of it as representing a row in a spreadsheet or a database. To give an example of a record data type, we will create a record for a shop inventory. The record contains the name of the item, a 40-character string, the number of items in the store represented by a 32-bit integer, and, finally, a price represented by a 32-bit float. These consecutive steps show how to create a record data type:

1. Create the record:

```
In: t = dtype([('name', str_, 40), ('numitems', int32), ('price', float32)])
In: t
Out: dtype([('name', '|S40'), ('numitems', '<i4'), ('price', '<f4')])
```

2. View the type (we can view the type of a field as well):

```
In: t['name']
Out: dtype('|S40')
```

If you don't give the `array()` function a data type, it will assume that it is dealing with floating point numbers. To create the array now, we really have to specify the data type; otherwise, we will get a `TypeError`:

```
In: itemz = array([('Meaning of life DVD', 42, 3.14), ('Butter', 13,
2.72)], dtype=t)
In: itemz[1]
Out: ('Butter', 13, 2.7200000286102295)
```

### **What just happened?**

We created a record data type, which is a heterogeneous data type. The record contained a name as a character string, a number as an integer, and a price represented by a float. The code for this example can be found in the `record.py` file in this book's code bundle.

## **One-dimensional slicing and indexing**

Slicing of one-dimensional NumPy arrays works just like slicing of Python lists. Select a piece of an array from index 3 to 7 that extracts the elements 3 through 6:

```
In: a = arange(9)
In: a[3:7]
Out: array([3, 4, 5, 6])
```

Select elements from index 0 to 7 with step 2 as follows:

```
In: a[7:2]
Out: array([0, 2, 4, 6])
```

Similarly, as in Python, use negative indices and reverse the array with this code snippet:

```
In: a[::-1]
Out: array([8, 7, 6, 5, 4, 3, 2, 1, 0])
```

### **Time for action – slicing and indexing multidimensional arrays**

The `ndarray` class supports slicing over multiple dimensions. For convenience, we refer to many dimensions at once, with an ellipsis.

1. To illustrate, create an array with the `arange()` function and reshape it:

```
In: b = arange(24).reshape(2,3,4)
In: b.shape
Out: (2, 3, 4)
```

```
In: b
Out:
array([[[ 0,  1,  2,  3],
        [ 4,  5,  6,  7],
        [ 8,  9, 10, 11]],
       [[12, 13, 14, 15],
        [16, 17, 18, 19],
        [20, 21, 22, 23]])
```

The array `b` has 24 elements with values 0 to 23 and we reshaped it to be a two-by-three-by-four, three-dimensional array. We can visualize this as a two-story building with 12 rooms on each floor, 3 rows and 4 columns (alternatively we can think of it as a spreadsheet with sheets, rows, and columns). As you have probably guessed, the `reshape()` function changes the shape of an array. We give it a tuple of integers, corresponding to the new shape. If the dimensions are not compatible with the data, an exception is thrown.

2. We can select a single room using its three coordinates, namely, the floor, column, and row. For example, the room on the first floor, in the first row, and in the first column (we can have floor 0 and room 0—it's just a matter of convention) can be represented by the following:

```
In: b[0,0,0]
Out: 0
```

3. If we don't care about the floor, but still want the first column and row, we replace the first index by `a:` (**colon**) because we just need to specify the floor number and omit the other indices:

```
In: b[:,0,0]
Out: array([ 0, 12])
```

Select the first floor in this code:

```
In: b[0]
Out:
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

We can also write this:

```
In: b[0, :, :]
Out:
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```



An ellipsis (...) replaces multiple colons, so, the preceding code is equivalent to this:

```
In: b[0, ...]
Out:
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

Furthermore, get the second row on the first floor:

```
In: b[0,1]
Out: array([4, 5, 6, 7])
```

- 4. Using steps to slice:** Furthermore, also select every second element of this selection:

```
In: b[0,1,::2]
Out: array([4, 6])
```

- 5. Using an ellipsis to slice:** If we want to select all the rooms on both floors that are in the second column, regardless of the row, type this code:

```
In: b[...,1]
Out:
array([[ 1,  5,  9],
       [13, 17, 21]])
```

Similarly, select all the rooms on the second row, regardless of floor and column, by writing the following code snippet:

```
In: b[:,1]
Out:
array([[ 4,  5,  6,  7],
       [16, 17, 18, 19]])
```

If we want to select rooms on the ground floor second column, then type this:

```
In: b[0,:,1]
Out: array([1, 5, 9])
```

- 6. Using negative indices:** If we want to select the first floor, last column, then type the following code snippet:

```
In: b[0,,-1]
Out: array([ 3,  7, 11])
```

If we want to select rooms on the ground floor, last column reversed, then type the following code snippet:

```
In: b[0,::-1, -1]
Out: array([11,  7,  3])
```

Select every second element of that slice as follows:

```
In: b[0,::2,-1]
Out: array([ 3, 11])
```

The command that reverses a one-dimensional array puts the top floor following the ground floor as follows:

```
In: b[::-1]
Out:
array([[12, 13, 14, 15],
       [16, 17, 18, 19],
       [20, 21, 22, 23]],
      [[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

### ***What just happened?***

We sliced a multidimensional NumPy array using several different methods. The code for this example can be found in the `slicing.py` file in this book's code bundle.

## **Time for action – manipulating array shapes**

We already learned about the `reshape()` function. Another recurring task is flattening of arrays. When we flatten multidimensional NumPy arrays, the result is a one-dimensional array with the same data.

- 1. Ravel:** Accomplish this with the `ravel()` function:

```
In: b
Out:
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]],
      [[12, 13, 14, 15],
       [16, 17, 18, 19],
       [20, 21, 22, 23]])

In: b.ravel()
Out:
array([ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11, 12, 13, 14,
       15, 16,
       17, 18, 19, 20, 21, 22, 23])
```

- 2. Flatten:** The appropriately named function, `flatten()` does the same as `ravel()`, but `flatten()` always allocates new memory whereas `ravel()` might return a view of the array. A view is a way to share an array, but you need to be careful with views because modifying the view affects the underlying array, and therefore this impacts other views. An array copy is safer; however, it uses more memory:

```
In: b.flatten()
Out:
array([ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11, 12, 13, 14,
       15, 16,
       17, 18, 19, 20, 21, 22, 23])
```

- 3. Setting the shape with a tuple:** Besides the `reshape()` function, we can also set the shape directly with a tuple, which is shown here:

```
In: b.shape = (6,4)
In: b
Out:
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11],
       [12, 13, 14, 15],
       [16, 17, 18, 19],
       [20, 21, 22, 23]])
```

As you can see, this changes the array directly. Now, we have a six-by-four array.

- 4. Transpose:** In linear algebra, it is common to transpose matrices.



Linear algebra is a branch of mathematics dealing among others with **matrices**. Matrices are the two-dimensional equivalent of vectors and contain numbers in a rectangular or square grid. Transposing a matrix entails flipping the matrix in such a manner that the matrix rows become the matrix columns and vice versa. Khan Academy has a course on linear algebra, which includes transposing matrices at [https://www.khanacademy.org/math/linear-algebra/matrix\\_transformations/matrix\\_transpose/v/linear-algebra-transpose-of-a-matrix](https://www.khanacademy.org/math/linear-algebra/matrix_transformations/matrix_transpose/v/linear-algebra-transpose-of-a-matrix).

We can do this too using the following code:

```
In: b.transpose()
Out:
array([[ 0,  4,  8, 12, 16, 20],
       [ 1,  5,  9, 13, 17, 21],
       [ 2,  6, 10, 14, 18, 22],
       [ 3,  7, 11, 15, 19, 23]])
```

- 5. Resize:** The `resize()` method works just like the `reshape()` function, but modifies the array it operates on:

```
In: b.resize((2,12))
In: b
Out:
array([[ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11],
       [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]])
```

### What just happened?

We manipulated the shapes of NumPy arrays using the `ravel()` function, the `flatten()` function, the `reshape()` function, and the `resize()` method, as explained in the following table:

Function	Description
<code>ravel()</code>	This function returns a one-dimensional array with the same data as the input array and doesn't always return a copy
<code>flatten()</code>	This is a method of <code>ndarray</code> , which flattens arrays and always returns a copy of the array
<code>reshape()</code>	This function modifies the shape of an array
<code>resize()</code>	This function changes the shape of an array and adds copies of the input array if necessary

The code for this example is in the `shapemanipulation.py` file in this book's code bundle.

## Stacking

Arrays can be stacked horizontally, depth wise, or vertically. We can use, for that purpose, the `vstack()`, `dstack()`, `hstack()`, `column_stack()`, `row_stack()`, and `concatenate()` functions.

### Time for action – stacking arrays

First, set up some arrays:

```
In: a = arange(9).reshape(3,3)
In: a
Out:
array([[0, 1, 2],
       [3, 4, 5],
       [6, 7, 8]])
In: b = 2 * a
```

```
In: b
Out:
array([[ 0,  2,  4],
       [ 6,  8, 10],
       [12, 14, 16]])
```

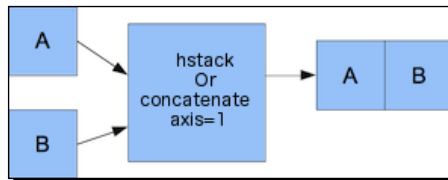
- 1. Horizontal stacking:** Starting with horizontal stacking, form a tuple of the ndarray objects and give it to the `hstack()` function as follows:

```
In: hstack((a, b))
Out:
array([[ 0,  1,  2,  0,  2,  4],
       [ 3,  4,  5,  6,  8, 10],
       [ 6,  7,  8, 12, 14, 16]])
```

Achieve the same with the `concatenate()` function as follows (the axis argument here is equivalent to axes in a Cartesian coordinate system and corresponds to the array dimensions):

```
In: concatenate((a, b), axis=1)
Out:
array([[ 0,  1,  2,  0,  2,  4],
       [ 3,  4,  5,  6,  8, 10],
       [ 6,  7,  8, 12, 14, 16]])
```

This image shows horizontal stacking with the `concatenate()` function:



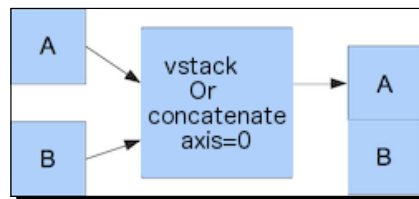
- 2. Vertical stacking:** With vertical stacking, again, a tuple is formed. This time, it is given to the `vstack()` function as follows:

```
In: vstack((a, b))
Out:
array([[ 0,  1,  2],
       [ 3,  4,  5],
       [ 6,  7,  8],
       [ 0,  2,  4],
       [ 6,  8, 10],
       [12, 14, 16]])
```

The `concatenate()` function produces the same result with the axis set to 0. This is the default value for the `axis` argument:

```
In: concatenate((a, b), axis=0)
Out:
array([[ 0,  1,  2],
       [ 3,  4,  5],
       [ 6,  7,  8],
       [ 0,  2,  4],
       [ 6,  8, 10],
       [12, 14, 16]])
```

The following diagram shows vertical stacking with `concatenate()` function:



- 3. Depth stacking:** Additionally, depth-wise stacking using `dstack()` and a tuple stacks a list of arrays along the third axis (depth). For instance, stack two-dimensional arrays of image data on top of each other:

```
In: dstack((a, b))
Out:
array([[[ 0,  0],
        [ 1,  2],
        [ 2,  4]],
       [[ 3,  6],
        [ 4,  8],
        [ 5, 10]],
       [[ 6, 12],
        [ 7, 14],
        [ 8, 16]])])
```

- 4. Column stacking:** Stack the one-dimensional arrays with the `column_stack()` function column-wise as follows:

```
In: oned = arange(2)
In: oned
Out: array([0, 1])
In: twice_oned = 2 * oned
In: twice_oned
Out: array([0, 2])
```

```
In: column_stack((oned, twice_oned))
Out:
array([[0, 0],
       [1, 2]])
```

Two-dimensional arrays are stacked the way `hstack()` stacks them:

```
In: column_stack((a, b))
Out:
array([[ 0,  1,  2,  0,  2,  4],
       [ 3,  4,  5,  6,  8, 10],
       [ 6,  7,  8, 12, 14, 16]])
In: column_stack((a, b)) == hstack((a, b))
Out:
array([[ True,  True,  True,  True,  True,  True],
       [ True,  True,  True,  True,  True,  True],
       [ True,  True,  True,  True,  True,  True]], dtype=bool)
```

Yes, you guessed it right! We compared two arrays with the `==` operator.



The `==` operator is used in Python to compare for equality. When applied to NumPy arrays, the operator performs element-wise comparisons. For more information about the Python comparison operators, have a look at <http://www.pythonlearn.com/html-009/book004.html>.

- 5. Row stacking:** NumPy, of course, also has a function that does row-wise stacking. It is called `row_stack()`, and, for one-dimensional arrays, it just stacks the arrays in rows into a two-dimensional array:

```
In: row_stack((oned, twice_oned))
Out:
array([[0, 1],
       [0, 2]])
```

The `row_stack()` function results for two-dimensional arrays are equal to, yes, exactly, the `vstack()` function results:

```
In: row_stack((a, b))
Out:
array([[ 0,  1,  2],
       [ 3,  4,  5],
       [ 6,  7,  8],
```

```

    [ 0,  2,  4],
    [ 6,  8, 10],
    [12, 14, 16]])
In: row_stack((a,b)) == vstack((a, b))
Out:
array([[ True,  True,  True],
       [ True,  True,  True],
       [ True,  True,  True],
       [ True,  True,  True],
       [ True,  True,  True],
       [ True,  True,  True]], dtype=bool)

```

### ***What just happened?***

We stacked arrays horizontally, depth wise, and vertically. We used the `vstack()`, `dstack()`, `hstack()`, `column_stack()`, `row_stack()`, and `concatenate()` functions as summarized in the following table:

Function	Description
<code>vstack()</code>	This function stacks arrays vertically
<code>dstack()</code>	This function stacks arrays depth-wise along the third axis
<code>hstack()</code>	This function stacks arrays horizontally
<code>column_stack()</code>	This function stacks one-dimensional arrays as columns to create a two-dimensional array
<code>row_stack()</code>	This function stacks array vertically
<code>concatenate()</code>	This function concatenates a list or a tuple of arrays

The code for this example is in the `stacking.py` file in this book's code bundle.

### **Splitting**

Arrays can be split vertically, horizontally, or depth wise. The functions involved are `hsplit()`, `vsplit()`, `dsplit()`, and `split()`. We can either split into arrays of the same shape or indicate the position after which the split should occur.



## Time for action – splitting arrays

The following steps demonstrate arrays splitting:

- 1. Horizontal splitting:** The ensuing code splits an array along its horizontal axis into three pieces of the same size and shape:

```
In: a
Out:
array([[0, 1, 2],
       [3, 4, 5],
       [6, 7, 8]])
In: hsplit(a, 3)
Out:
[array([[0],
       [3],
       [6]]),
 array([[1],
       [4],
       [7]]),
 array([[2],
       [5],
       [8]])]
```

Compare it with a call of the `split()` function, with extra parameter `axis=1`:

```
In: split(a, 3, axis=1)
Out:
[array([[0],
       [3],
       [6]]),
 array([[1],
       [4],
       [7]]),
 array([[2],
       [5],
       [8]])]
```

- 2. Vertical splitting:** `vsplit()` splits along the vertical axis:

```
In: vsplit(a, 3)
Out: [array([[0, 1, 2]]), array([[3, 4, 5]]), array([[6, 7, 8]])]
```

The `split()` function, with `axis=0`, also splits along the vertical axis:

```
In: split(a, 3, axis=0)
Out: [array([[0, 1, 2]]), array([[3, 4, 5]]), array([[6, 7, 8]])]
```

- 3. Depth-wise splitting:** The `dsplit()` function, unsurprisingly, splits depth-wise. Create an array of rank 3 first before splitting:

```
In: c = arange(27).reshape(3, 3, 3)
```

```
In: c
```

```
Out:
```

```
array([[[ 0,  1,  2],
        [ 3,  4,  5],
        [ 6,  7,  8]],
       [[ 9, 10, 11],
        [12, 13, 14],
        [15, 16, 17]],
       [[18, 19, 20],
        [21, 22, 23],
        [24, 25, 26]]])
```

```
In: dsplit(c, 3)
```

```
Out:
```

```
[array([[[ 0],
         [ 3],
         [ 6]],
       [[ 9],
        [12],
        [15]],
       [[18],
        [21],
        [24]]]),
 array([[[ 1],
         [ 4],
         [ 7]],
       [[10],
        [13],
        [16]],
       [[19],
        [22],
        [25]]]),
 array([[[ 2],
         [ 5],
         [ 8]],
       [[11],
        [14],
        [17]],
       [[20],
        [23],
        [26]]])]
```

## What just happened?

We split arrays using the `hsplit()`, `vsplit()`, `dsplit()`, and `split()` functions. These functions differ in the axis along which the split occurs. The code for this example is in the `splitting.py` file in this book's code bundle.

## Array attributes

Besides the `shape` and `dtype` attributes, `ndarray` has a number of other attributes, as shown in the following list:

- ◆ The `ndim` attribute gives the number of dimensions:  

```
In: b
Out:
array([[ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11],
       [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]])
In: b.ndim
Out: 2
```
- ◆ The `size` attribute contains the number of elements. This is shown as follows:  

```
In: b.size
Out: 24
```
- ◆ The `itemsize` attribute gives the number of bytes for each element in the array:  

```
In: b.itemsize
Out: 8
```
- ◆ If you want the total number of bytes the array requires, you can have a look at `nbytes`. This is just a product of the `itemsize` and `size` attributes:  

```
In: b.nbytes
Out: 192
In: b.size * b.itemsize
Out: 192
```
- ◆ The `T` attribute has the same effect as the `transpose()` function, which is shown as follows:  

```
In: b.resize(6,4)
In: b
Out:
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11],
       [12, 13, 14, 15],
       [16, 17, 18, 19],
```

```

        [20, 21, 22, 23]])
In: b.T
Out:
array([[ 0,  4,  8, 12, 16, 20],
       [ 1,  5,  9, 13, 17, 21],
       [ 2,  6, 10, 14, 18, 22],
       [ 3,  7, 11, 15, 19, 23]])

```

- ◆ If the array has a rank lower than 2, we will just get a view of the array:

```

In: b.ndim
Out: 1
In: b.T
Out: array([0, 1, 2, 3, 4])

```

Complex numbers in NumPy are represented by `j`. For example, create an array with complex numbers as in the following code:

```

In: b = array([1.j + 1, 2.j + 3])
In: b
Out: array([ 1.+1.j,  3.+2.j])

```

- ◆ The `real` attribute gives us the real part of the array, or the array itself if it only contains real numbers:

```

In: b.real
Out: array([ 1.,  3.])

```

- ◆ The `imag` attribute contains the imaginary part of the array:

```

In: b.imag
Out: array([ 1.,  2.])

```

- ◆ If the array contains complex numbers, then the data type is automatically also complex:

```

In: b.dtype
Out: dtype('complex128')
In: b.dtype.str
Out: '<c16'

```

- ◆ The `flat` attribute returns a `numpy.flatiter` object. This is the only way to acquire a `flatiter`—we do not have access to a `flatiter` constructor. The `flat` iterator enables us to loop through an array as if it is a flat array, as shown in the following example:

```

In: b = arange(4).reshape(2,2)
In: b
Out:
array([[0, 1],
       [2, 3]])

```

```
In: f = b.flat
In: f
Out: <numpy.flatiter object at 0x103013e00>
In: for item in f: print item
     .....:
     0
     1
     2
     3
```

It is possible to get an element directly with the `flatiter` object:

```
In: b.flat[2]
Out: 2
```

And, it is also possible to directly get multiple elements:

```
In: b.flat[[1,3]]
Out: array([1, 3])
```

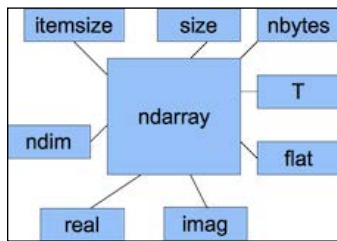
The `flat` attribute is settable. Setting the value of the `flat` attribute leads to overwriting the values of the whole array:

```
In: b.flat = 7
In: b
Out:
array([[7, 7],
       [7, 7]])
```

Or, it can also lead to overwriting the values of selected elements:

```
In: b.flat[[1,3]] = 1
In: b
Out:
array([[7, 1],
       [7, 1]])
```

The following diagram shows the different types of attributes of the `ndarray` class:



## Time for action – converting arrays

Convert a NumPy array to a Python list with the `tolist()` function:

1. Convert to a list:

```
In: b
Out: array([ 1.+1.j,  3.+2.j])
In: b.tolist()
Out: [(1+1j), (3+2j)]
```

2. The `astype()` function converts the array to an array of the specified type:

```
In: b
Out: array([ 1.+1.j,  3.+2.j])
In: b.astype(int)
/usr/local/bin/ipython:1: ComplexWarning: Casting complex values
to real discards the imaginary part
  #!/usr/bin/python
Out: array([1, 3])
```



We are losing the imaginary part when casting from the NumPy complex type (not the plain vanilla Python one) to `int`. The `astype()` function also accepts the name of a type as a string.

```
In: b.astype('complex')
Out: array([ 1.+1.j,  3.+2.j])
```

It won't show any warning this time because we used the proper data type.

### What just happened?

We converted NumPy arrays to a list and to arrays of different data types. The code for this example is in the `arrayconversion.py` file in this book's code bundle.

## Summary

In this chapter, you learned a lot about NumPy fundamentals: data types and arrays. Arrays have several attributes describing them. You learned that one of these attributes is the data type, which, in NumPy, is represented by a fully-fledged object.

NumPy arrays can be sliced and indexed in an efficient manner, just like Python lists. NumPy arrays have the added ability of working with multiple dimensions.

The shape of an array can be manipulated in many ways—stacking, resizing, reshaping, and splitting. A great number of convenience functions for shape manipulation were demonstrated in this chapter.

Having learned about the basics, it's time to move on to the study of commonly used functions in *Chapter 3, Getting Familiar with Commonly Used Functions*, which includes basic statistical and mathematical functions.

# 3

## Getting Familiar with Commonly Used Functions

*In this chapter, we will have a look at common NumPy functions. In particular, we will learn how to load data from files by using an example involving historical stock prices. Also, we will get to see the basic NumPy mathematical and statistical functions.*

*We will learn how to read from and write to files. Also, we will get a taste of the functional programming and linear algebra possibilities in NumPy.*

In this chapter, we shall cover the following topics:

- ◆ Functions working on arrays
- ◆ Loading arrays from files
- ◆ Writing arrays to files
- ◆ Simple mathematical and statistical functions

### File I/O

First, we will learn about file I/O with NumPy. Data is usually stored in files. You would not get far if you were not able to read from and write to files.



## Time for action – reading and writing files

As an example of file I/O, we will create an identity matrix and store its contents in a file.



In this and other chapters, we will use the following line by convention to import NumPy:

```
import numpy as np
```

Perform the following steps to do so:

1. The identity matrix is a square matrix with ones on the main diagonal and zeros for the rest (see <https://www.khanacademy.org/math/precalculus/precalc-matrices/zero-identity-matrix-tutorial/v/identity-matrix>).

The identity matrix can be created with the `eye()` function. The only argument that we need to give the `eye()` function is the number of ones. So, for instance, for a two-by-two matrix, write the following code:

```
i2 = np.eye(2)
print(i2)
```

The output is:

```
[[ 1.  0.]
 [ 0.  1.]]
```

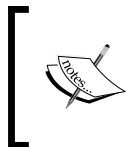
2. Save the data in a plain text file with the `savetxt()` function. Specify the name of the file that we want to save the data in and the array containing the data itself:

```
np.savetxt("eye.txt", i2)
```

A file called `eye.txt` should have been created in the same directory as the Python script.

### What just happened?

Reading and writing files is a necessary skill for data analysis. We wrote to a file with `savetxt()`. We made an identity matrix with the `eye()` function.



Instead of a filename, we can also provide a **file handle**. A file handle is a term in many programming languages, which means a variable pointing to a file, like a postal address. For more information on how to get a file handle in Python, please refer to <http://www.diveintopython3.net/files.html>.

You can check for yourself whether the contents are as expected. The code for this example can be downloaded from the book support website: <https://www.packtpub.com/books/content/support> (see `save.py`)

```
import numpy as np

i2 = np.eye(2)
print(i2)

np.savetxt("eye.txt", i2)
```

## Comma-separated value files

Files in the **Comma-separated value (CSV)** format are encountered quite frequently. Often, the CSV file is just a dump from a database. Usually, each field in the CSV file corresponds to a database table column. As we all know, spreadsheet programs, such as Excel, can produce CSV files, as well.

### Time for action – loading from CSV files

How do we deal with CSV files? Luckily, the `loadtxt()` function can conveniently read CSV files, split up the fields, and load the data into NumPy arrays. In the following example, we will load historical stock price data for Apple (the company, not the fruit). The data is in CSV format and is part of the code bundle for this book. The first column contains a symbol that identifies the stock. In our case, it is AAPL. Second is the date in dd-mm-yyyy format. The third column is empty. Then, in order, we have the open, high, low, and close price. Last, but not least, is the trading volume of the day. This is what a line looks like:

```
AAPL,28-01-2011, ,344.17,344.4,333.53,336.1,21144800
```

For now, we are only interested in the close price and volume. In the preceding sample, that will be 336.1 and 21144800. Store the close price and volume in two arrays as follows:

```
c,v=np.loadtxt('data.csv', delimiter=',', usecols=(6,7), unpack=True)
```

As you can see, data is stored in the `data.csv` file. We have set the delimiter to `(comma)`, since we are dealing with a CSV file. The `usecols` parameter is set through a tuple to get the seventh and eighth fields, which correspond to the close price and volume. The `unpack` argument is set to `True`, which means that data will be unpacked and assigned to the `c` and `v` variables that will hold the close price and volume, respectively.

## Volume Weighted Average Price

**Volume Weighted Average Price (VWAP)** is a very important quantity in finance. It represents an average price for a financial asset (see <https://www.khanacademy.org/math/probability/descriptive-statistics/old-stats-videos/v/statistics-the-average>). The higher the volume, the more significant a price move typically is. VWAP is often used in algorithmic trading and is calculated using volume values as weights.

### Time for action – calculating Volume Weighted Average Price

The following are the actions that we will take:

1. Read the data into arrays.
2. Calculate VWAP:

```
from __future__ import print_function
import numpy as np
c,v=np.loadtxt('data.csv', delimiter=',', usecols=(6,7),
unpack=True)
vwap = np.average(c, weights=v)
print("VWAP =", vwap)
```

The output is as follows:

```
VWAP = 350.589549353
```

### What just happened?

That wasn't very hard, was it? We just called the `average()` function and set its `weights` parameter to use the `v` array for weights. By the way, NumPy also has a function to calculate the arithmetic mean. This is an unweighted average with all the weights equal to 1.

### The mean() function

The `mean()` function is quite friendly and not so mean. This function calculates the arithmetic mean of an array.

The arithmetic mean is given by the following formula:

$$\frac{1}{n} \sum_{i=1}^n a_i$$



It sums the values in an array `a` and divides the sum by the number of elements `n` (see [https://www.khanacademy.org/math/probability/descriptive-statistics/central\\_tendency/e/mean\\_median\\_and\\_mode](https://www.khanacademy.org/math/probability/descriptive-statistics/central_tendency/e/mean_median_and_mode)).

Let's see it in action:

```
print("mean =", np.mean(c))
```

As a result, we get the following printout:

```
mean = 351.037666667
```

## Time-weighted average price

In finance, **time-weighted average price (TWAP)** is another average price measure. Now that we are at it, let's compute the TWAP too. It is just a variation on a theme really. The idea is that recent price quotes are more important, so we should give recent prices higher weights. The easiest way is to create an array with the `arange()` function of increasing values from zero to the number of elements in the close price array. This is not necessarily the correct way. In fact, most of the examples concerning stock price analysis in this book are only illustrative. The following is the TWAP code:

```
t = np.arange(len(c))
print("twap =", np.average(c, weights=t))
```

It produces the following output:

```
twap = 352.428321839
```

The TWAP is even higher than the mean.

## Pop quiz – computing the weighted average

Q1. Which function returns the weighted average of an array?

1. weighted average
2. waverage
3. average
4. avg

## Have a go hero – calculating other averages

Try doing the same calculation using the open price. Calculate the mean for the volume and the other prices.

## Value range

Usually, we don't only want to know the average or arithmetic mean of a set of values, which are in the middle, to know we also want the extremes, the full range—the highest and lowest values. The sample data that we are using here already has those values per day—the high and low price. However, we need to know the highest value of the high price and the lowest price value of the low price.

## Time for action – finding highest and lowest values

The `min()` and `max()` functions are the answer for our requirement. Perform the following steps to find the highest and lowest values:

1. First, read our file again and store the values for the high and low prices into arrays:

```
h,l=np.loadtxt('data.csv', delimiter=',', usecols=(4,5),
unpack=True)
```

The only thing that changed is the `usecols` parameter, since the high and low prices are situated in different columns.

2. The following code gets the price range:

```
print("highest =", np.max(h))
print("lowest =", np.min(l))
```

These are the values returned:

```
highest = 364.9
lowest = 333.53
```

Now, it's easy to get a midpoint, so it is left as an exercise for you to attempt.

3. NumPy allows us to compute the spread of an array with a function called `ptp()`. The `ptp()` function returns the difference between the maximum and minimum values of an array. In other words, it is equal to `max(array) - min(array)`. Call the `ptp()` function:

```
print("Spread high price", np.ptp(h))
print("Spread low price", np.ptp(l))
```

You will see this text printed:

```
Spread high price 24.86
```

```
Spread low price 26.97
```

## ***What just happened?***

We defined a range of highest to lowest values for the price. The highest value was given by applying the `max()` function to the high price array. Similarly, the lowest value was found by calling the `min()` function to the low price array. We also calculated the peak-to-peak distance with the `ptp()` function:

```
from __future__ import print_function
import numpy as np

h,l=np.loadtxt('data.csv', delimiter=',', usecols=(4,5), unpack=True)
print("highest =", np.max(h))
print("lowest =", np.min(l))
print((np.max(h) + np.min(l)) /2)

print("Spread high price", np.ptp(h))
print("Spread low price", np.ptp(l))
```

## **Statistics**

Stock traders are interested in the most probable close price. Common sense says that this should be close to some kind of an average as the price dances around a mean, due to random fluctuations. The arithmetic mean and weighted average are ways to find the center of a distribution of values. However, neither are robust and both are sensitive to outliers. Outliers are extreme values that are much bigger or smaller than the typical values in a dataset. Usually, outliers are caused by a rare phenomenon or a measurement error. For instance, if we have a close price value of a million dollars, this will influence the outcome of our calculations.

### **Time for action – performing simple statistics**

We can use some kind of threshold to weed out outliers, but there is a better way. It is called the median, and it basically picks the middle value of a sorted set of values (see [https://www.khanacademy.org/math/probability/descriptive-statistics/central\\_tendency/e/mean\\_median\\_and\\_mode](https://www.khanacademy.org/math/probability/descriptive-statistics/central_tendency/e/mean_median_and_mode)). One half of the data is below the median and the other half is above it. For example, if we have the values of 1, 2, 3, 4, and 5, then the median will be 3, since it is in the middle.

These are the steps to calculate the median:

1. Create a new Python script and call it `simplestats.py`. You already know how to load the data from a CSV file into an array. So, copy that line of code and make sure that it only gets the close price. The code should appear like this:

```
c=np.loadtxt('data.csv', delimiter=',', usecols=(6,), unpack=True)
```

2. The function that will do the magic for us is called `median()`. We will call it and print the result immediately. Add the following line of code:

```
print("median =", np.median(c))
```

The program prints the following output:

```
median = 352.055
```

3. Since it is our first time using the `median()` function, we would like to check whether this is correct. Obviously, we can do it by just going through the file and finding the correct value, but that is no fun. Instead, we will just mimic the median algorithm by sorting the close price array and printing the middle value of the sorted array. The `msort()` function does the first part for us. Call the function, store the sorted array, and then print it:

```
sorted_close = np.msort(c)
print("sorted =", sorted_close)
```

This prints the following output:

```
sorted = [ 336.1  338.61  339.32  342.62  342.88  343.44  344.32  345.03  346.5
346.67  348.16  349.31  350.56  351.88  351.99  352.12  352.47  353.21
354.54  355.2  355.36  355.76  356.85  358.16  358.3  359.18  359.56
359.9  360.  363.13]
```

Yup, it works! Let's now get the middle value of the sorted array:

```
N = len(c)
print "middle =", sorted[(N - 1)/2]
```

The preceding snippet gives us the following output:

```
middle = 351.99
```

4. Hey, that's a different value than the one the `median()` function gave us. How come? Upon further investigation, we find that the `median()` function return value doesn't even appear in our file. That's even stranger! Before filing bugs with the NumPy team, let's have a look at the documentation:

```
$ python
>>> import numpy as np
>>> help(np.median)
```

This mystery is easy to solve. It turns out that our naive algorithm only works for arrays with odd lengths. For even-length arrays, the median is calculated from the average of the two array values in the middle. Therefore, type the following code:

```
print("average middle =", (sorted[N / 2] + sorted[(N - 1) / 2]) / 2)
```

This prints the following output:

```
average middle = 352.055
```

5. Another statistical measure that we are concerned with is variance. Variance tells us how much a variable varies (see [https://www.khanacademy.org/math/probability/descriptive-statistics/variance\\_std\\_deviation/e/variance](https://www.khanacademy.org/math/probability/descriptive-statistics/variance_std_deviation/e/variance)). In our case, it also tells us how risky an investment is, since a stock price that varies too wildly is bound to get us into trouble.


Calculate the variance of the close price (with NumPy, this is just a one-liner):

```
print("variance =", np.var(c))
```

This gives us the following output:

```
variance = 50.1265178889
```

6. Not that we don't trust NumPy or anything, but let's double-check using the definition of variance, as found in the documentation. Mind you, this definition might be different than the one in your statistics book, but that is quite common in the field of statistics.



The population variance is defined as the mean of the square of deviations from the mean, divided by the number of elements in the array:

$$\frac{1}{n} \sum_{i=1}^n (a_i - mean)^2$$

Some books tell us to divide by the number of elements in the array minus one (this is called a **sample variance**):

```
print("variance from definition =", np.mean((c - c.mean())**2))
```

The output is as follows:

```
variance from definition = 50.1265178889
```



## What just happened?

Maybe you noticed something new. We suddenly called the `mean()` function on the `c` array. Yes, this is legal, because the `ndarray` class has a `mean()` method. This is for your convenience. For now, just keep in mind that this is possible. The code for this example can be found in `simplestats.py`:

```
from __future__ import print_function
import numpy as np

c=np.loadtxt('data.csv', delimiter=',', usecols=(6,), unpack=True)
print("median =", np.median(c))
sorted = np.msort(c)
print("sorted =", sorted)

N = len(c)
print("middle =", sorted[(N - 1)/2])
print("average middle =", (sorted[N / 2] + sorted[(N - 1) / 2]) / 2)

print("variance =", np.var(c))
print("variance from definition =", np.mean((c - c.mean())**2))
```

## Stock returns

In academic literature, it is more common to base analysis on stock returns and log returns of the close price. Simple returns are just the rate of change from one value to the next. Logarithmic returns, or log returns, are determined by taking the log of all the prices and calculating the differences between them. In high school, we learned that:

$$\log(a) - \log(b) = \log\left(\frac{a}{b}\right)$$

Log returns, therefore, also measure the rate of change. Returns are dimensionless, since, in the act of dividing, we divide dollar by dollar (or some other currency). Anyway, investors are most likely to be interested in the variance or standard deviation of the returns, as this represents risk.

## Time for action – analyzing stock returns

Perform the following steps to analyze stock returns:

1. First, let's calculate simple returns. NumPy has the `diff()` function that returns an array that is built up of the difference between two consecutive array elements. This is sort of like differentiation in calculus (the derivative of price with respect to time). To get the returns, we also have to divide by the value of the previous day. We must be careful though. The array returned by `diff()` is one element shorter than the close prices array. After careful deliberation, we get the following code:

```
returns = np.diff( arr ) / arr[ : -1]
```

Notice that we don't use the last value in the divisor. The standard deviation is equal to the square root of variance. Compute the standard deviation using the `std()` function:

```
print("Standard deviation =", np.std(returns))
```

This results in the following output:

```
Standard deviation = 0.0129221344368
```

2. The log return or logarithmic return is even easier to calculate. Use the `log()` function to get the natural logarithm of the close price and then unleash the `diff()` function on the result:

```
logreturns = np.diff(np.log(c))
```

Normally, we have to check that the input array doesn't have zeros or negative numbers. If it does, we will get an error. Stock prices are, however, always positive, so we didn't have to check.

3. Quite likely, we will be interested in days when the return is positive. In the current setup, we can get the next best thing with the `where()` function, which returns the indices of an array that satisfies a condition. Just type the following code:

```
posretindices = np.where(returns > 0)
print("Indices with positive returns", posretindices)
```

This gives us a number of indices for the array elements that are positive as a tuple, recognizable by the round brackets on both sides of the printout:

```
Indices with positive returns (array([ 0,  1,  4,  5,  6,  7,  9,
 10, 11, 12, 16, 17, 18, 19, 21, 22, 23, 25, 28]),)
```

4. In investing, volatility measures price variation of a financial security. Historical volatility is calculated from historical price data. The logarithmic returns are interesting if you want to know the historical volatility—for instance, the annualized or monthly volatility. The annualized volatility is equal to the standard deviation of the log returns as a ratio of its mean, divided by one over the square root of the number of business days in a year, usually one assumes 252. Calculate it with the `std()` and `mean()` functions, as in the following code:

```
annual_volatility = np.std(logreturns)/np.mean(logreturns)
annual_volatility = annual_volatility / np.sqrt(1./252.)
print(annual_volatility)
```

Take notice of the division within the `sqrt()` function. Since, in Python, integer division works differently than float division, we needed to use floats to make sure that we get the proper results. The monthly volatility is similarly given by the following code:

```
print("Monthly volatility", annual_volatility * np.sqrt(1./12.))
```

## ***What just happened?***

We calculated the simple stock returns with the `diff()` function, which calculates differences between sequential elements. The `log()` function computes the natural logarithms of array elements. We used it to calculate the logarithmic returns. At the end of this section, we calculated the annual and monthly volatility (see `returns.py`):

```
from __future__ import print_function
import numpy as np

c=np.loadtxt('data.csv', delimiter=',', usecols=(6,), unpack=True)

returns = np.diff( c ) / c[ : -1]
print("Standard deviation =", np.std(returns))

logreturns = np.diff( np.log(c) )

posretindices = np.where(returns > 0)
print("Indices with positive returns", posretindices)

annual_volatility = np.std(logreturns)/np.mean(logreturns)
annual_volatility = annual_volatility / np.sqrt(1./252.)
print("Annual volatility", annual_volatility)

print("Monthly volatility", annual_volatility * np.sqrt(1./12.))
```

## Dates

Do you sometimes have the Monday blues or Friday fever? Ever wondered whether the stock market suffers from these phenomena? Well, I think this certainly warrants extensive research.

### Time for action – dealing with dates

First, we will read the close price data. Second, we will split the prices according to the day of the week. Third, for each weekday, we will calculate the average price. Finally, we will find out which day of the week has the highest average and which has the lowest average. A word of warning before we commence: you might be tempted to use the result to buy stock on one day and sell on the other. However, we don't have enough data to make this kind of decisions.

Coders hate dates because they are so complicated! NumPy is very much oriented toward floating point operations. For this reason, we need to take extra effort to process dates. Try it out yourself; put the following code in a script or use the one that comes with this book:

```
dates, close=np.loadtxt('data.csv', delimiter=',',
                       usecols=(1,6), unpack=True)
```

Execute the script and the following error will appear:

```
ValueError: invalid literal for float(): 28-01-2011
```

Now, perform the following steps to deal with dates:

1. Obviously, NumPy tried to convert the dates into floats. What we have to do is tell NumPy explicitly how to convert the dates. The `loadtxt()` function has a special parameter for this purpose. The parameter is called `converters` and is a dictionary that links columns with the so-called converter functions. It is our responsibility to write the converter function. Write the function down:

```
# Monday 0
# Tuesday 1
# Wednesday 2
# Thursday 3
# Friday 4
# Saturday 5
# Sunday 6
def datestr2num(s):
    return datetime.datetime.strptime(s, "%d-%m-%Y").date().
weekday()
```

We give the `datestr2num()` function dates as a string, such as 28-01-2011. The string is first turned into a `datetime` object, using a specified format `%d-%m-%Y`. By the way, this is standard Python and is not related to NumPy itself (see <https://docs.python.org/2/library/datetime.html#strptime-and-strptime-behavior>). Second, the `datetime` object is turned into a day. Finally, the `weekday` method is called on the date to return a number. As you can read in the comments, the number is between 0 and 6. 0 is, for instance, Monday, and 6 is Sunday. The actual number, of course, is not important for our algorithm; it is only used as identification.

**2.** Now, hook up our date converter function:

```
dates, close=np.loadtxt('data.csv', delimiter=',', usecols=(1,6),
converters={1: datestr2num}, unpack=True)
print "Dates =", dates
```

This prints the following output:

```
Dates = [ 4.  0.  1.  2.  3.  4.  0.  1.  2.  3.  4.  0.  1.  2.
 3.  4.  1.  2.  4.  0.  1.  2.  3.  4.  0.  1.  2.  3.  4.]
```

No Saturdays and Sundays, as you can see. Exchanges are closed over the weekend.

**3.** We will now make an array that has five elements for each day of the week. Initialize the values of the array to 0:

```
averages = np.zeros(5)
```

This array will hold the averages for each weekday.

**4.** We already learned about the `where()` function that returns indices of the array for elements that conform to a specified condition. The `take()` function can use these indices and takes the values of the corresponding array items. We will use the `take()` function to get the close prices for each weekday. In the following loop, we go through the date values 0 to 4, better known as Monday to Friday. We get the indices with the `where()` function for each day and store it in the `indices` array. Then, we retrieve the values corresponding to the indices, using the `take()` function. Finally, compute an average for each weekday and store it in the `averages` array, like this:

```
for i in range(5):
    indices = np.where(dates == i)
    prices = np.take(close, indices)
    avg = np.mean(prices)
    print("Day", i, "prices", prices, "Average", avg)
    averages[i] = avg
```

The loop prints the following output:

```
Day 0 prices [[ 339.32  351.88  359.18  353.21  355.36]] Average
351.79
Day 1 prices [[ 345.03  355.2   359.9   338.61  349.31  355.76]]
Average 350.635
Day 2 prices [[ 344.32  358.16  363.13  342.62  352.12  352.47]]
Average 352.136666667
Day 3 prices [[ 343.44  354.54  358.3   342.88  359.56  346.67]]
Average 350.898333333
Day 4 prices [[ 336.1   346.5   356.85  350.56  348.16  360.
351.99]] Average 350.022857143
```

5. If you want, you can go ahead and find out which day has the highest average, and which the lowest. However, it is just as easy to find this out with the `max()` and `min()` functions, as shown here:

```
top = np.max(averages)
print("Highest average", top)
print("Top day of the week", np.argmax(averages))
bottom = np.min(averages)
print("Lowest average", bottom)
print("Bottom day of the week", np.argmin(averages))
```

The output is as follows:

```
Highest average 352.136666667
Top day of the week 2
Lowest average 350.022857143
Bottom day of the week 4
```

## ***What just happened?***

The `argmin()` function returned the index of the lowest value in the `averages` array. The index returned was 4, which corresponds to Friday. The `argmax()` function returned the index of the highest value in the `averages` array. The index returned was 2, which corresponds to Wednesday (see `weekdays.py`):

```
from __future__ import print_function
import numpy as np
from datetime import datetime

# Monday 0
# Tuesday 1
# Wednesday 2
```

```
# Thursday 3
# Friday 4
# Saturday 5
# Sunday 6
def datestr2num(s):
    return datetime.strptime(s, "%d-%m-%Y").date().weekday()

dates, close=np.loadtxt('data.csv', delimiter=',', usecols=(1,6),
converters={1: datestr2num}, unpack=True)
print("Dates =", dates)

averages = np.zeros(5)

for i in range(5):
    indices = np.where(dates == i)
    prices = np.take(close, indices)
    avg = np.mean(prices)
    print("Day", i, "prices", prices, "Average", avg)
    averages[i] = avg

top = np.max(averages)
print("Highest average", top)
print("Top day of the week", np.argmax(averages))

bottom = np.min(averages)
print("Lowest average", bottom)
print("Bottom day of the week", np.argmin(averages))
```

## **Have a go hero – looking at VWAP and TWAP**

Hey, that was fun! For the sample data, it appears that Friday is the cheapest day and Wednesday is the day when your Apple stock will be worth the most. Ignoring the fact that we have very little data, is there a better method to compute the averages? Shouldn't we involve volume data as well? Maybe it makes more sense to you to do a time-weighted average. Give it a go! Calculate the VWAP and TWAP. You can find some hints on how to go about doing this at the beginning of this chapter.

## Time for action – using the datetime64 data type

The `datetime64` data type was introduced in NumPy 1.7.0 (see <http://docs.scipy.org/doc/numpy/reference/arrays.datetime.html>).

1. To learn about the `datetime64` data type, start a Python shell and import NumPy as follows:

```
$ python
>>> import numpy as np
```

Create a `datetime64` from a string (you can use another date if you like):

```
>>> np.datetime64('2015-04-22')
numpy.datetime64('2015-04-22')
```

In the preceding code, we created a `datetime64` for April 22, 2015, which happens to be Earth Day. We used the YYYY-MM-DD format, where Y corresponds to the year, M corresponds to the month, and D corresponds to the day of the month. NumPy uses the ISO 8601 standard (see [http://en.wikipedia.org/wiki/ISO\\_8601](http://en.wikipedia.org/wiki/ISO_8601)). This is an international standard to represent dates and times. ISO 8601 allows the YYYY-MM-DD, YYYY-MM, and YYYYMMDD formats. Check for yourself, as follows:

```
>>> np.datetime64('2015-04-22')
numpy.datetime64('2015-04-22')
>>> np.datetime64('2015-04')
numpy.datetime64('2015-04')
```

2. By default, ISO 8601 uses the local time zone. Times can be specified using the format T[hh:mm:ss]. For example, define January 1, 1677 at 8:19 p.m. as follows:

```
>>> local = np.datetime64('1677-01-01T20:19')
>>> local
numpy.datetime64('1677-01-01T20:19Z')
```

Additionally, a string in the format [hh:mm] specifies an offset that is relative to the UTC time zone. Create a `datetime64` with 9 hours offset, as follows:

```
>>> with_offset = np.datetime64('1677-01-01T20:19-0900')
>>> with_offset
numpy.datetime64('1677-01-02T05:19Z')
```

The Z at the end stands for Zulu time, which is how UTC is sometimes referred to.

Subtract the two `datetime64` objects from each other:

```
>>> local - with_offset
numpy.timedelta64(-540, 'm')
```



The subtraction creates a NumPy `timedelta64` object, which in this case, indicates a 540 minute difference. We can also add or subtract a number of days to a `datetime64` object. For instance, April 22, 2015 happens to be a Wednesday. With the `arange()` function, create an array holding all the Wednesdays from April 22, 2015 until May 22, 2015 as follows:

```
>>> np.arange('2015-04-22', '2015-05-22', 7, dtype='datetime64')
array(['2015-04-22', '2015-04-29', '2015-05-06', '2015-05-13',
      '2015-05-20'], dtype='datetime64 [D]')
```

Note that in this case, it is mandatory to specify the `dtype` argument, otherwise NumPy thinks that we are dealing with strings.

### ***What just happened?***

We learned about the NumPy `datetime64` type. This data type allows us to manipulate dates and times with ease. Its features include simple arithmetic and creation of arrays using the normal NumPy capabilities.

## **Weekly summary**

The data that we used in the previous *Time for action* section is end-of-day data. In essence, it is summarized data compiled from the trade data for a certain day. If you are interested in the market and have decades of data, you might want to summarize and compress the data even further. Let's summarize the data of Apple stocks to give us weekly summaries.

### **Time for action – summarizing data**

The data we will summarize will be for a whole business week, running from Monday to Friday. During the period covered by the data, there was one holiday on February 21, President's Day. This happened to be a Monday and the US stock exchanges were closed on this day. As a consequence, there is no entry for this day, in the sample. The first day in the sample is a Friday, which is inconvenient. Use the following instructions to summarize data:

- 1.** To simplify, just have a look at the first three weeks in the sample— later, you can have a go at improving this:

```
close = close[:16]
dates = dates[:16]
```

We will be building on the code from the previous *Time for action* section.

- Commencing, we will find the first Monday in our sample data. Recall that Mondays have the code 0 in Python. This is what we will put in the condition of the `where()` function. Then, we will need to extract the first element that has index 0. The result will be a multidimensional array. Flatten this with the `ravel()` function:

```
# get first Monday
first_monday = np.ravel(np.where(dates == 0))[0]
print("The first Monday index is", first_monday)
```

This will print the following output:

```
The first Monday index is 1
```

- The next logical step is to find the Friday before last Friday in the sample. The logic is similar to the one for finding the first Monday, and the code for Friday is 4. Additionally, we are looking for the second to last element with index 2:

```
# get last Friday
last_friday = np.ravel(np.where(dates == 4))[-2]
print("The last Friday index is", last_friday)
```

This will give us the following output:

```
The last Friday index is 15
```

- Next, create an array with the indices of all the days in the three weeks:

```
weeks_indices = np.arange(first_monday, last_friday + 1)
print("Weeks indices initial", weeks_indices)
```

- Split the array in pieces of size 5 with the `split()` function:

```
weeks_indices = np.split(weeks_indices, 3)
print("Weeks indices after split", weeks_indices)
```

This splits the array as follows:

```
Weeks indices after split [array([1, 2, 3, 4, 5]), array([ 6,  7,
8,  9, 10]), array([11, 12, 13, 14, 15])]
```

- In NumPy, array dimensions are called **axes**. Now, we will get fancy with the `apply_along_axis()` function. This function calls another function, which we will provide, to operate on each of the elements of an array. Currently, we have an array with three elements. Each array item corresponds to one week in our sample and contains indices of the corresponding items. Call the `apply_along_axis()` function by supplying the name of our function, called `summarize()`, which we will define shortly. Furthermore, specify the axis or dimension number (such as 1), the array to operate on, and a variable number of arguments for the `summarize()` function, if any:

```
weeksummary = np.apply_along_axis(summarize, 1,
    weeks_indices, open, high, low, close)
print("Week summary", weeksummary)
```

7. For each week, the `summarize()` function returns a tuple that holds the open, high, low, and close price for the week, similar to end-of-day data:

```
def summarize(a, o, h, l, c):
    monday_open = o[a[0]]
    week_high = np.max( np.take(h, a) )
    week_low = np.min( np.take(l, a) )
    friday_close = c[a[-1]]

    return("APPL", monday_open, week_high,
           week_low, friday_close)
```

Notice that we used the `take()` function to get the actual values from indices. Calculating the high and low values for the week was easily done with the `max()` and `min()` functions. The open for the week is the open for the first day in the week—Monday. Likewise, the close is the close for the last day of the week—Friday:

```
Week summary [['APPL' '335.8' '346.7' '334.3' '346.5']
              ['APPL' '347.89' '360.0' '347.64' '356.85']
              ['APPL' '356.79' '364.9' '349.52' '350.56']]
```

8. Store the data in a file with the NumPy `savetxt()` function:

```
np.savetxt("weeksummary.csv", weeksummary, delimiter=",",
           fmt="%s")
```

As you can see, have specified a filename, the array we want to store, a delimiter (in this case a comma), and the format we want to store floating point numbers in.

The format string starts with a percent sign. Second is an optional flag. The `-flag` means left justify, `0` means left pad with zeros, and `+` means precede with `+` or `-`. Third is an optional width. The width indicates the minimum number of characters. Fourth, a dot is followed by a number linked to precision. Finally, there comes a character specifier; in our example, the character specifier is a string. The character codes are described as follows:

Character code	Description
c	character
d or i	signed decimal integer
e or E	scientific notation with e or E.
f	decimal floating point
g,G	use the shorter of e,E or f
o	signed octal

Character code	Description
s	string of characters
u	unsigned decimal integer
x,X	unsigned hexadecimal integer

View the generated file in your favorite editor or type at the command line:

```
$ cat weeksummary.csv
APPL,335.8,346.7,334.3,346.5
APPL,347.89,360.0,347.64,356.85
APPL,356.79,364.9,349.52,350.56
```

### ***What just happened?***

We did something that is not even possible in some programming languages. We defined a function and passed it as an argument to the `apply_along_axis()` function.



The programming paradigm described here is called functional programming. You can read more about functional programming in Python at <https://docs.python.org/2/howto/functional.html>.

Arguments for the `summarize()` function were neatly passed by `apply_along_axis()` (see `weeksummary.py`):

```
from __future__ import print_function
import numpy as np
from datetime import datetime

# Monday 0
# Tuesday 1
# Wednesday 2
# Thursday 3
# Friday 4
# Saturday 5
# Sunday 6
def datestr2num(s):
    return datetime.strptime(s, "%d-%m-%Y").date().weekday()

dates, open, high, low, close=np.loadtxt('data.csv', delimiter=',',
usecols=(1, 3, 4, 5, 6), converters={1: datestr2num}, unpack=True)
close = close[:16]
dates = dates[:16]
```

### Getting Familiar with Commonly Used Functions

---

```
# get first Monday
first_monday = np.ravel(np.where(dates == 0))[0]
print("The first Monday index is", first_monday)

# get last Friday
last_friday = np.ravel(np.where(dates == 4))[-1]
print("The last Friday index is", last_friday)

weeks_indices = np.arange(first_monday, last_friday + 1)
print("Weeks indices initial", weeks_indices)

weeks_indices = np.split(weeks_indices, 3)
print("Weeks indices after split", weeks_indices)

def summarize(a, o, h, l, c):
    monday_open = o[a[0]]
    week_high = np.max( np.take(h, a) )
    week_low = np.min( np.take(l, a) )
    friday_close = c[a[-1]]

    return("APPL", monday_open, week_high, week_low, friday_close)

weeksummary = np.apply_along_axis(summarize, 1, weeks_indices, open,
high, low, close)
print("Week summary", weeksummary)

np.savetxt("weeksummary.csv", weeksummary, delimiter=",", fmt="%s")
```

### Have a go hero – improving the code

Change the code to deal with a holiday. Time the code to see how big the speedup due to `apply_along_axis()` is.

## Average True Range

The **Average True Range (ATR)** is a technical indicator that measures volatility of stock prices. The ATR calculation is not important further but will serve as an example of several NumPy functions, including the `maximum()` function.

## Time for action – calculating the Average True Range

To calculate the ATR, perform the following steps:

1. The ATR is based on the low and high price of  $N$  days, usually the last 20 days.

```
N = 5
h = h[-N:]
l = l[-N:]
```

2. We also need to know the close price of the previous day:

```
previousclose = c[-N - 1: -1]
```

For each day, we calculate the following:

The daily range—the difference between the high and low price:

```
h - l
```

The difference between the high and previous close:

```
h - previousclose
```

The difference between the previous close and the low price:

```
previousclose - l
```

3. The `max()` function returns the maximum of an array. Based on those three values, we calculate the so-called true range, which is the maximum of these values. We are now interested in the element-wise maxima across arrays—meaning the maxima of the first elements in the arrays, the second elements in the arrays, and so on. Use the NumPy `maximum()` function instead of the `max()` function for this purpose:

```
truerange = np.maximum(h - l, h - previousclose, previousclose - l)
```

4. Create an `atr` array of size  $N$  and initialize its values to 0:

```
atr = np.zeros(N)
```

5. The first value of the array is just the average of the `truerange` array:

```
atr[0] = np.mean(truerange)
```

Calculate the other values with the following formula:

$$\frac{((N-1)PATR + TR)}{N}$$

Here, PATR is the previous day's ATR; TR is the true range:

```
for i in range(1, N):
    atr[i] = (N - 1) * atr[i - 1] + truerange[i]
    atr[i] /= N
```

## ***What just happened?***

We formed three arrays, one for each of the three ranges—daily range, the gap between the high of today and the close of yesterday, and the gap between the close of yesterday and the low of today. This tells us how much the stock price moved and, therefore, how volatile it is. The algorithm requires us to find the maximum value for each day. The `max()` function that we used before can give us the maximum value within an array, but that is not what we want here. We need the maximum value across arrays, so we want the maximum value of the first elements in the three arrays, the second elements, and so on. In preceding *Time for action* section, we saw that the `maximum()` function can do this. After this, we computed a moving average of the true range values (see `atr.py`):

```
from __future__ import print_function
import numpy as np

h, l, c = np.loadtxt('data.csv', delimiter=',', usecols=(4, 5, 6),
                    unpack=True)

N = 5
h = h[-N:]
l = l[-N:]

print("len(h)", len(h), "len(l)", len(l))
print("Close", c)
previousclose = c[-N - 1: -1]

print("len(previousclose)", len(previousclose))
print("Previous close", previousclose)
truerange = np.maximum(h - l, h - previousclose, previousclose - l)

print("True range", truerange)

atr = np.zeros(N)

atr[0] = np.mean(truerange)
```

```

for i in range(1, N):
    atr[i] = (N - 1) * atr[i - 1] + truerange[i]
    atr[i] /= N

print("ATR", atr)

```

In the following sections, we will learn better ways to calculate moving averages.

### Have a go hero – taking the minimum() function for a spin

Besides the `maximum()` function, there is a `minimum()` function. You can probably guess what it does. Make a small script or start an interactive session in IPython to test your assumptions.

## Simple Moving Average

The **Simple Moving Average (SMA)** is commonly used to analyze time-series data. To calculate it, we define a moving window of  $N$  periods,  $N$  days in our case. We move this window along the data and calculate the mean of the values inside the window.

### Time for action – computing the Simple Moving Average

The moving average is easy enough to compute with a few loops and the `mean()` function, but NumPy has a better alternative—the `convolve()` function. The SMA is, after all, nothing more than a convolution with equal weights or, if you like, unweighted.



**Convolution** is a mathematical operation on two functions defined as the integral of the product of the two functions after one of the functions is reversed and shifted.

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$$

Convolution is described on Wikipedia at <https://en.wikipedia.org/wiki/Convolution>. Khan Academy also has a tutorial on convolution at <https://www.khanacademy.org/math/differential-equations/laplace-transform/convolution-integral/v/introduction-to-the-convolution>.



Use the following steps to compute the SMA:

1. Use the `ones()` function to create an array of size `N` and elements initialized to 1, and then, divide the array by `N` to give us the weights:

```
N = 5
weights = np.ones(N) / N
print("Weights", weights)
```

For `N = 5`, this gives us the following output:

```
Weights [ 0.2  0.2  0.2  0.2  0.2]
```

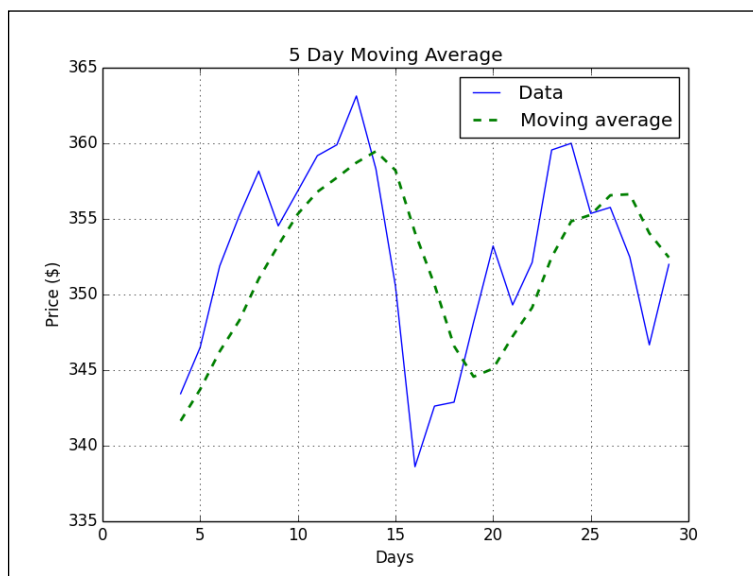
2. Now, call the `convolve()` function with these weights:

```
c = np.loadtxt('data.csv', delimiter=',', usecols=(6,),
unpack=True)
sma = np.convolve(weights, c)[N-1:-N+1]
```

3. From the array returned by `convolve()`, we extracted the data in the center of size `N`. The following code makes an array of time values and plots with `matplotlib` that we will cover in a later chapter:

```
c = np.loadtxt('data.csv', delimiter=',', usecols=(6,),
unpack=True)
sma = np.convolve(weights, c)[N-1:-N+1]
t = np.arange(N - 1, len(c))
plt.plot(t, c[N-1:], lw=1.0, label="Data")
plt.plot(t, sma, '--', lw=2.0, label="Moving average")
plt.title("5 Day Moving Average")
plt.xlabel("Days")
plt.ylabel("Price ($)")
plt.grid()
plt.legend()
plt.show()
```

In the following chart, the smooth dashed line is the 5 day SMA and the jagged thin line is the close price:



### ***What just happened?***

We computed the SMA for the close stock price. It turns out that the SMA is just a signal processing technique—a convolution with weights  $1/N$ , where  $N$  is the size of the moving average window. We learned that the `ones()` function can create an array with ones and the `convolve()` function calculates the convolution of a dataset with specified weights (see `sma.py`):

```
from __future__ import print_function
import numpy as np
import matplotlib.pyplot as plt

N = 5

weights = np.ones(N) / N
print("Weights", weights)

c = np.loadtxt('data.csv', delimiter=',', usecols=(6,), unpack=True)
sma = np.convolve(weights, c)[N-1:-N+1]
t = np.arange(N - 1, len(c))
plt.plot(t, c[N-1:], lw=1.0, label="Data")
plt.plot(t, sma, '--', lw=2.0, label="Moving average")
```

```
plt.title("5 Day Moving Average")
plt.xlabel("Days")
plt.ylabel("Price ($)")
plt.grid()
plt.legend()
plt.show()
```

## Exponential Moving Average

The **Exponential Moving Average (EMA)** is a popular alternative to the SMA. This method uses exponentially decreasing weights. The weights for points in the past decrease exponentially but never reach zero. We will learn about the `exp()` and `linspace()` functions while calculating the weights.

### Time for action – calculating the Exponential Moving Average

Given an array, the `exp()` function calculates the exponential of each array element. For example, look at the following code:

```
x = np.arange(5)
print("Exp", np.exp(x))
```

It gives the following output:

```
Exp [ 1.          2.71828183  7.3890561  20.08553692  54.59815003]
```

The `linspace()` function takes as parameters a start value, a stop value, and optionally an array size. It returns an array of evenly spaced numbers. This is an example:

```
print("Linspace", np.linspace(-1, 0, 5))
```

This will give us the following output:

```
Linspace [-1.  -0.75 -0.5  -0.25  0. ]
```

Calculate the EMA for our data:

1. Now, back to the weights, calculate them with `exp()` and `linspace()`:  

```
N = 5
weights = np.exp(np.linspace(-1., 0., N))
```
2. Normalize the weights with the `ndarray sum()` method:  

```
weights /= weights.sum()
print("Weights", weights)
```

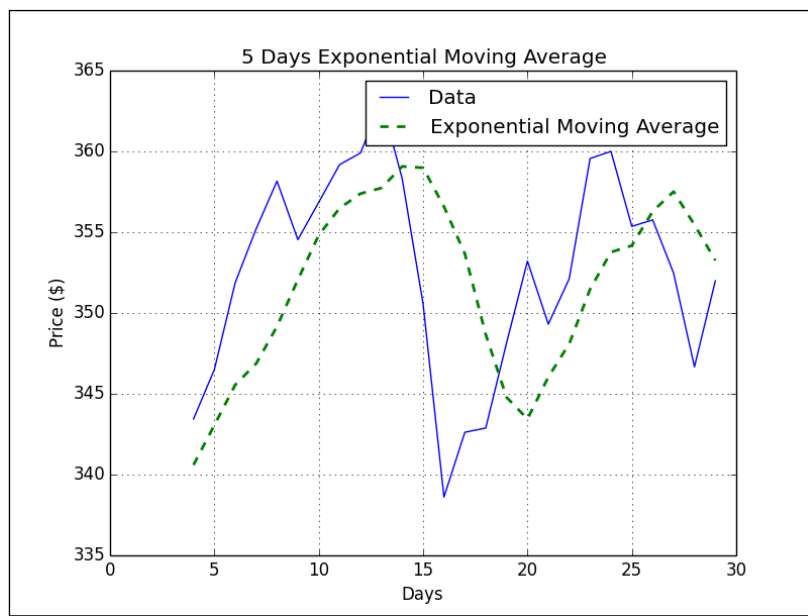
For  $N = 5$ , we get these weights:

```
Weights [ 0.11405072  0.14644403  0.18803785  0.24144538
0.31002201]
```

3. After this, use the `convolve()` function that we learned about in the SMA section and also plot the results:

```
c = np.loadtxt('data.csv', delimiter=',', usecols=(6,),
unpack=True)
ema = np.convolve(weights, c)[N-1:-N+1]
t = np.arange(N - 1, len(c))
plt.plot(t, c[N-1:], lw=1.0, label='Data')
plt.plot(t, ema, '--', lw=2.0, label='Exponential Moving Average')
plt.title('5 Days Exponential Moving Average')
plt.xlabel('Days')
plt.ylabel('Price ($)')
plt.legend()
plt.grid()
plt.show()
```

This gives us a nice chart where, again, the close price is the thin jagged line and the EMA is the smooth dashed line:



## What just happened?

We calculated the EMA of the close price. First, we computed exponentially decreasing weights with the `exp()` and `linspace()` functions. The `linspace()` function gave us an array with evenly spaced elements, and, then, we calculated the exponential for these numbers. We called the `ndarray sum()` method in order to normalize the weights. After this, we applied the `convolve()` trick that we learned in the SMA section (see `ema.py`):

```
from __future__ import print_function
import numpy as np
import matplotlib.pyplot as plt

x = np.arange(5)
print("Exp", np.exp(x))
print("Linspace", np.linspace(-1, 0, 5))

# Calculate weights
N = 5
weights = np.exp(np.linspace(-1., 0., N))

# Normalize weights
weights /= weights.sum()
print("Weights", weights)

c = np.loadtxt('data.csv', delimiter=',', usecols=(6,), unpack=True)
ema = np.convolve(weights, c)[N-1:-N+1]
t = np.arange(N - 1, len(c))
plt.plot(t, c[N-1:], lw=1.0, label='Data')
plt.plot(t, ema, '--', lw=2.0, label='Exponential Moving Average')
plt.title('5 Days Exponential Moving Average')
plt.xlabel('Days')
plt.ylabel('Price ($)')
plt.legend()
plt.grid()
plt.show()
```

## Bollinger Bands

**Bollinger Bands** are yet another technical indicator. Yes, there are thousands of them. This one is named after its inventor and indicates a range for the price of a financial security. It consists of three parts:

1. A Simple Moving Average.

2. An upper band of two standard deviations above this moving average—the standard deviation is derived from the same data with which the moving average is calculated.
3. A lower band of two standard deviations below the moving average.

## Time for action – enveloping with Bollinger Bands

We already know how to calculate the SMA. So, if you need to refresh your memory, please review the *Time for action – computing the simple average* section in this chapter. This example will introduce the NumPy `fill()` function. The `fill()` function sets the value of an array to a scalar value. The function should be faster than `array.flat = scalar` or setting the values of the array one-by-one in a loop. Perform the following steps to envelope with the Bollinger Bands:

1. Starting with an array called `sma` that contains the moving average values, we will loop through all the datasets corresponding to those values. After forming the dataset, calculate the standard deviation. Note that at a certain point, it will be necessary to calculate the difference between each data point and the corresponding average value. If we do not have NumPy, we will loop through these points and subtract each of the values one-by-one from the corresponding average. However, the NumPy `fill()` function allows us to construct an array that has elements set to the same value. This enables us to save on one loop and subtract arrays in one go:

```
deviation = []
C = len(c)

for i in range(N - 1, C):
    if i + N < C:
        dev = c[i: i + N]
    else:
        dev = c[-N:]

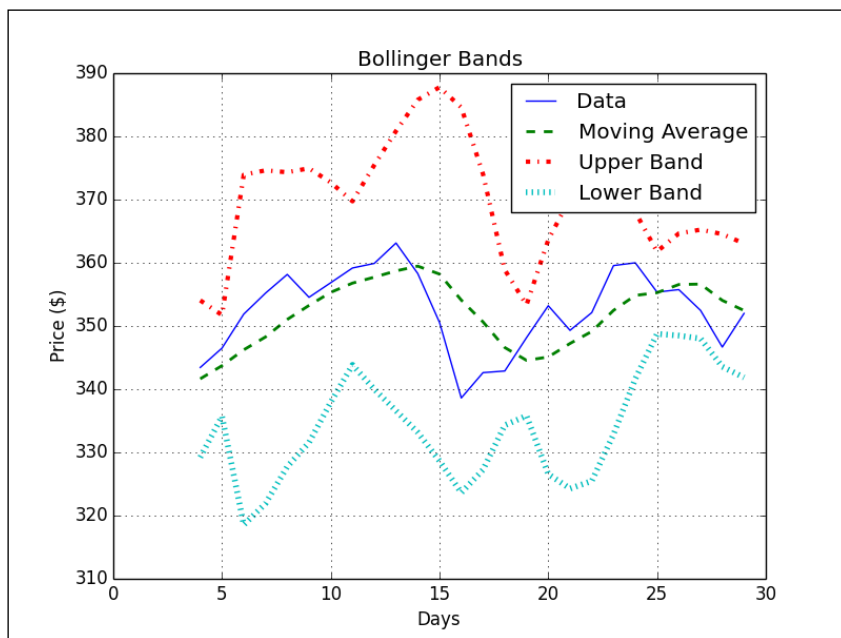
    averages = np.zeros(N)
    averages.fill(sma[i - N - 1])
    dev = dev - averages
    dev = dev ** 2
    dev = np.sqrt(np.mean(dev))
    deviation.append(dev)

deviation = 2 * np.array(deviation)
print(len(deviation), len(sma))
upperBB = sma + deviation
lowerBB = sma - deviation
```

2. To plot, we will use the following code (don't worry about it now; we will see how this works in *Chapter 9, Plotting with matplotlib*):

```
t = np.arange(N - 1, C)
plt.plot(t, c_slice, lw=1.0, label='Data')
plt.plot(t, sma, '--', lw=2.0, label='Moving Average')
plt.plot(t, upperBB, '-.', lw=3.0, label='Upper Band')
plt.plot(t, lowerBB, ':', lw=4.0, label='Lower Band')
plt.title('Bollinger Bands')
plt.xlabel('Days')
plt.ylabel('Price ($)')
plt.grid()
plt.legend()
plt.show()
```

Following is a chart showing the Bollinger Bands for our data. The jagged thin line in the middle represents the close price, and the dashed, smoother line crossing it is the moving average:



## ***What just happened?***

We worked out the Bollinger Bands that envelope the close price of our data. More importantly, we got acquainted with the NumPy `fill()` function. This function fills an array with a scalar value. This is the only parameter of the `fill()` function (see `bollingerbands.py`):

```

from __future__ import print_function
import numpy as np
import matplotlib.pyplot as plt

N = 5

weights = np.ones(N) / N
print("Weights", weights)

c = np.loadtxt('data.csv', delimiter=',', usecols=(6,), unpack=True)
sma = np.convolve(weights, c)[N-1:-N+1]
deviation = []
C = len(c)

for i in range(N - 1, C):
    if i + N < C:
        dev = c[i: i + N]
    else:
        dev = c[-N:]

    averages = np.zeros(N)
    averages.fill(sma[i - N - 1])
    dev = dev - averages
    dev = dev ** 2
    dev = np.sqrt(np.mean(dev))
    deviation.append(dev)

deviation = 2 * np.array(deviation)
print(len(deviation), len(sma))
upperBB = sma + deviation
lowerBB = sma - deviation

c_slice = c[N-1:]
between_bands = np.where((c_slice < upperBB) & (c_slice > lowerBB))

print(lowerBB[between_bands])
print(c[between_bands])
print(upperBB[between_bands])

```



```
between_bands = len(np.ravel(between_bands))
print("Ratio between bands", float(between_bands)/len(c_slice))

t = np.arange(N - 1, C)
plt.plot(t, c_slice, lw=1.0, label='Data')
plt.plot(t, sma, '--', lw=2.0, label='Moving Average')
plt.plot(t, upperBB, '-.', lw=3.0, label='Upper Band')
plt.plot(t, lowerBB, ':', lw=4.0, label='Lower Band')
plt.title('Bollinger Bands')
plt.xlabel('Days')
plt.ylabel('Price ($)')
plt.grid()
plt.legend()
plt.show()
```

## Have a go hero – switching to Exponential Moving Average

It is customary to choose the SMA to center the Bollinger Band on. The second most popular choice is the EMA, so try that as an exercise. You can find a suitable example in this chapter, if you need pointers.

Check whether the `fill()` function is faster or is as fast as `array.flat = scalar`, or setting the value in a loop.

## Linear model


Many phenomena in science have a related linear relationship model. The NumPy `linalg` package deals with linear algebra computations. We will begin with the assumption that a price value can be derived from  $N$  previous prices based on a linear relationship relation.

## Time for action – predicting price with a linear model

Keeping an open mind, let's assume that we can express a stock price  $p$  as a linear combination of previous values, that is, a sum of those values multiplied by certain coefficients we need to determine:

$$p_t = b + \sum_{i=1}^N a_{t-i} p_{t-i}$$

In linear algebra terms, this boils down to finding a least-squares method (see [https://www.khanacademy.org/math/linear-algebra/alternate\\_bases/orthogonal\\_projections/v/linear-algebra-least-squares-approximation](https://www.khanacademy.org/math/linear-algebra/alternate_bases/orthogonal_projections/v/linear-algebra-least-squares-approximation)).



Independently of each other, the astronomers Legendre and Gauss created the least squares method around 1805 (see [http://en.wikipedia.org/wiki/Least\\_squares](http://en.wikipedia.org/wiki/Least_squares)). The method was initially used to analyze the motion of celestial bodies. The algorithm minimizes the sum of the squared residuals (the difference between measured and predicted values):

$$\sum_{i=1}^n (\text{measured}_i - \text{predicted}_i)^2$$

The recipe goes as follows:

1. First, form a vector `b` containing `N` price values:

```
b = c[-N:]
b = b[::-1]
print("b", x)
```

The result is as follows:

```
b [ 351.99  346.67  352.47  355.76  355.36]
```

2. Second, pre-initialize the matrix `A` to be `N`-by-`N` and contain zeros:

```
A = np.zeros((N, N), float)
Print("Zeros N by N", A)
```

The following should be printed on your screen:

```
Zeros N by N [[ 0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.]]
```

3. Third, fill the matrix `A` with `N` preceding price values for each value in `b`:

```
for i in range(N):
    A[i, ] = c[-N - 1 - i: - 1 - i]

print("A", A)
```

Now, `A` looks like this:

```
A [[ 360.    355.36  355.76  352.47  346.67]
   [ 359.56  360.    355.36  355.76  352.47]
   [ 352.12  359.56  360.    355.36  355.76]
   [ 349.31  352.12  359.56  360.    355.36]
   [ 353.21  349.31  352.12  359.56  360.  ]]
```

- 4.** The objective is to determine the coefficients that satisfy our linear model by solving the least squares problem. Employ the `lstsq()` function of the NumPy `linalg` package to do this:

```
(x, residuals, rank, s) = np.linalg.lstsq(A, b)
```

```
print(x, residuals, rank, s)
```

The result is as follows:

```
[ 0.78111069 -1.44411737  1.63563225 -0.89905126  0.92009049]
[] 5 [ 1.77736601e+03  1.49622969e+01  8.75528492e+00
5.15099261e+00  1.75199608e+00]
```

The tuple returned contains the coefficient `x` that we were after, an array comprising residuals, the rank of matrix `A`, and the singular values of `A`.

- 5.** Once we have the coefficients of our linear model, we can predict the next price value. Compute the dot product (with the NumPy `dot()` function) of the coefficients and the last known `N` prices:

```
print(np.dot(b, x))
```

The dot product (see [https://www.khanacademy.org/math/linear-algebra/vectors\\_and\\_spaces/dot\\_cross\\_products/v/vector-dot-product-and-vector-length](https://www.khanacademy.org/math/linear-algebra/vectors_and_spaces/dot_cross_products/v/vector-dot-product-and-vector-length)) is the linear combination of the coefficients `b` and the prices `x`. As a result, we get:

```
357.939161015
```

I looked it up; the actual close price of the next day was 353.56. So, our estimate with `N = 5` was not that far off.

## What just happened?

We predicted tomorrow's stock price today. If this works in practice, we can retire early! See, this book was a good investment, after all! We designed a linear model for the predictions. The financial problem was reduced to a linear algebraic one. NumPy's `linalg` package has a practical `lstsq()` function that helped us with the task at hand, estimating the coefficients of a linear model. After obtaining a solution, we plugged the numbers in the NumPy `dot()` function that presented us an estimate through linear regression (see `linearmodel.py`):

```
from __future__ import print_function
import numpy as np

N = 5

c = np.loadtxt('data.csv', delimiter=',', usecols=(6,), unpack=True)

b = c[-N:]
b = b[:, :-1]
print("b", b)

A = np.zeros((N, N), float)
print("Zeros N by N", A)

for i in range(N):
    A[i, ] = c[-N - 1 - i: - 1 - i]

print("A", A)

(x, residuals, rank, s) = np.linalg.lstsq(A, b)

print(x, residuals, rank, s)

print(np.dot(b, x))
```

## Trend lines

A trend line is a line among a number of the so-called pivot points on a stock chart. As the name suggests, the line's trend portrays the trend of the price development. In the past, traders drew trend lines on paper but nowadays, we can let a computer draw it for us. In this section, we shall use a very simple approach that probably won't be very useful in real life, but should clarify the principle well.

## Time for action – drawing trend lines

Perform the following steps to draw trend lines:

1. First, we need to determine the pivot points. We shall pretend they are equal to the arithmetic mean of the high, low, and close price:

```
h, l, c = np.loadtxt('data.csv', delimiter=',', usecols=(4, 5, 6), unpack=True)
```

```
pivots = (h + l + c) / 3
print("Pivots", pivots)
```

From the pivots, we can deduce the so-called **resistance** and **support levels**.

The support level is the lowest level at which the price rebounds. The resistance level is the highest level at which the price bounces back. These are not natural phenomena, they are merely estimates. Based on these estimates, it is possible to draw support and resistance trend lines. We will define the daily spread to be the difference of the high and low price.

2. Define a function to fit line to data to a line where  $y = at + b$ . The function should return  $a$  and  $b$ . This is another opportunity to apply the `lstsq()` function of the NumPy `linalg` package. Rewrite the line equation to  $y = Ax$ , where  $A = [t \ 1]$  and  $x = [a \ b]$ . Form  $A$  with the NumPy `ones_like()`, which creates an array, where all the values are equal to 1, using an input array as a template for the array dimensions:

```
def fit_line(t, y):
    A = np.vstack([t, np.ones_like(t)]).T
    return np.linalg.lstsq(A, y)[0]
```

3. Assuming that support levels are one daily spread below the pivots, and that **resistance levels** are one daily spread above the pivots, fit the support and resistance trend lines:

```
t = np.arange(len(c))
sa, sb = fit_line(t, pivots - (h - l))
ra, rb = fit_line(t, pivots + (h - l))
support = sa * t + sb
resistance = ra * t + rb
```

4. At this juncture, we have all the necessary information to draw the trend lines; however, it is wise to check how many points fall between the support and resistance levels. Obviously, if only a small percentage of the data is between the trend lines, then this setup is of no use to us. Make up a condition for points between the bands and select with the `where()` function, based on the following condition:

```
condition = (c > support) & (c < resistance)
print("Condition", condition)
between_bands = np.where(condition)
```

These are the printed condition values:

```
Condition [False False  True  True  True  True  True False False
 True False False
  False False  True False False False  True  True  True  True
 False False  True  True  True False  True]
```

Double-check the values:

```
print(support[between_bands])
print(c[between_bands])
print(resistance[between_bands])
```

The array returned by the `where()` function has rank 2, so call the `ravel()` function before calling the `len()` function:

```
between_bands = len(np.ravel(between_bands))
print("Number points between bands", between_bands)
print("Ratio between bands", float(between_bands)/len(c))
```

You will get the following result:

```
Number points between bands 15
Ratio between bands 0.5
```

As an extra bonus, we gained a predictive model. Extrapolate the next day resistance and support levels:

```
print("Tomorrows support", sa * (t[-1] + 1) + sb)
print("Tomorrows resistance", ra * (t[-1] + 1) + rb)
```

This results in the following output:

```
Tomorrows support 349.389157088
Tomorrows resistance 360.749340996
```

Another approach to figure out how many points are between the support and resistance estimates is to use `[]` and `intersect1d()`. Define selection criteria in the `[]` operator and intersect the results with the `intersect1d()` function:

```
a1 = c[c > support]
a2 = c[c < resistance]
print("Number of points between bands 2nd approach" ,len(np.
intersect1d(a1, a2)))
```

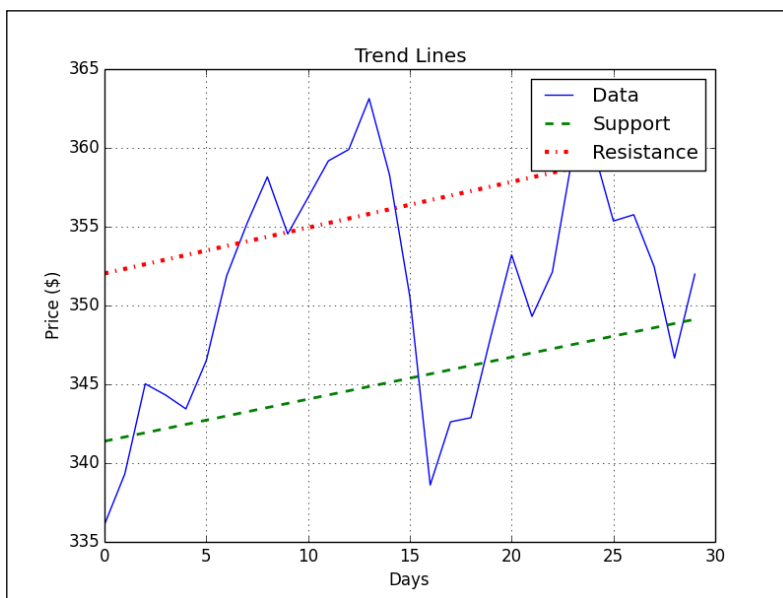
Not surprisingly, we get:

```
Number of points between bands 2nd approach 15
```

**5.** Once more, plot the results:

```
plt.plot(t, c, label='Data')
plt.plot(t, support, '--', lw=2.0, label='Support')
plt.plot(t, resistance, '-.', lw=3.0, label='Resistance')
plt.title('Trend Lines')
plt.xlabel('Days')
plt.ylabel('Price ($)')
plt.grid()
plt.legend()
plt.show()
```

In the following plot, we have the price data and the corresponding support and resistance lines:



## What just happened?

We drew trend lines without having to mess around with rulers, pencils, and paper charts. We defined a function that can fit data to a line with the NumPy `vstack()`, `ones_like()`, and `lstsq()` functions. We fit the data in order to define support and resistance trend lines. Then, we figured out how many points are within the support and resistance range. We did this using two separate methods that produced the same result.

The first method used the `where()` function with a Boolean condition. The second method made use of the `[]` operator and the `intersect1d()` function. The `intersect1d()` function returns an array of common elements from two arrays (see `trendline.py`):

```
from __future__ import print_function
import numpy as np
import matplotlib.pyplot as plt

def fit_line(t, y):
    ''' Fits t to a line y = at + b '''
    A = np.vstack([t, np.ones_like(t)]).T

    return np.linalg.lstsq(A, y)[0]

# Determine pivots
h, l, c = np.loadtxt('data.csv', delimiter=',', usecols=(4, 5, 6),
unpack=True)

pivots = (h + l + c) / 3
print("Pivots", pivots)

# Fit trend lines
t = np.arange(len(c))
sa, sb = fit_line(t, pivots - (h - l))
ra, rb = fit_line(t, pivots + (h - l))

support = sa * t + sb
resistance = ra * t + rb
condition = (c > support) & (c < resistance)
print("Condition", condition)
between_bands = np.where(condition)
print(support[between_bands])
print(c[between_bands])
print(resistance[between_bands])
between_bands = len(np.ravel(between_bands))
```



```
print("Number points between bands", between_bands)
print("Ratio between bands", float(between_bands)/len(c))

print("Tomorrows support", sa * (t[-1] + 1) + sb)
print("Tomorrows resistance", ra * (t[-1] + 1) + rb)

a1 = c[c > support]
a2 = c[c < resistance]
print("Number of points between bands 2nd approach" ,len(np.
intersect1d(a1, a2)))

# Plotting
plt.plot(t, c, label='Data')
plt.plot(t, support, '--', lw=2.0, label='Support')
plt.plot(t, resistance, '-.', lw=3.0, label='Resistance')
plt.title('Trend Lines')
plt.xlabel('Days')
plt.ylabel('Price ($)')
plt.grid()
plt.legend()
plt.show()
```

## Methods of ndarray

The NumPy `ndarray` class has a lot of methods that work on the array. Most of the time, these methods return an array. You may have noticed that many of the functions part of the NumPy library have a counterpart with the same name and functionality in the `ndarray` class. This is mostly due to the historical development of NumPy.

The list of `ndarray` methods is pretty long, so we cannot cover them all. The `mean()`, `var()`, `sum()`, `std()`, `argmax()`, `argmin()`, and `mean()` functions that we saw earlier are also `ndarray` methods.

### Time for action – clipping and compressing arrays

Here are a few examples of `ndarray` methods. Perform the following steps to clip and compress arrays:

1. The `clip()` method returns a clipped array, so that all values above a maximum value are set to the maximum and values below a minimum are set to the minimum value. Clip an array with values 0 to 4 to 1 and 2:

```
a = np.arange(5)
print("a =", a)
print("Clipped", a.clip(1, 2))
```

This gives the following output:

```
a = [0 1 2 3 4]
Clipped [1 1 2 2 2]
```

2. The `ndarray.compress()` method returns an array based on a condition. For instance, look at the following code:

```
a = np.arange(4)
print(a)
print("Compressed", a.compress(a > 2))
```

This returns the following output:

```
[0 1 2 3]
Compressed [3]
```

### ***What just happened?***

We created an array with values 0 to 3 and selected the last element with the `compress()` function based on the `a > 2` condition.

## **Factorial**

Many programming books have an example of calculating the factorial. We should not break with this tradition.

### **Time for action – calculating the factorial**

The `ndarray` class has the `prod()` method, which computes the product of the elements in an array. Perform the following steps to calculate the factorial:

1. Calculate the factorial of 8. To do this, generate an array with values 1 to 8 and call the `prod()` function on it:

```
b = np.arange(1, 9)
print("b =", b)
print("Factorial", b.prod())
```

Check the result with your pocket calculator:

```
b = [1 2 3 4 5 6 7 8]
Factorial 40320
```

This is nice, but what if we want to know all the factorials from 1 to 8?

2. No problem! Call the `cumprod()` method, which computes the cumulative product of an array:

```
print("Factorials", b.cumprod())
```

It's pocket calculator time again:

```
Factorials [ 1 2 6 24 120 720 5040 40320]
```

## What just happened?

We used the `prod()` and `cumprod()` functions to calculate factorials (see `ndarraymethods.py`):

```
from __future__ import print_function
import numpy as np

a = np.arange(5)
print("a =", a)
print("Clipped", a.clip(1, 2))

a = np.arange(4)
print(a)
print("Compressed", a.compress(a > 2))

b = np.arange(1, 9)
print("b =", b)
print("Factorial", b.prod())

print("Factorials", b.cumprod())
```

## Missing values and Jackknife resampling

Data often misses values because of errors or technical issues. Even if we are not missing values, we may have cause to suspect certain values. Once we doubt data values, derived values such as the arithmetic mean, which we learned to calculate in this chapter, become questionable too. It is common for these reasons to try to estimate how reliable the arithmetic mean, variance, and standard deviation are.

A simple but effective method is called **Jackknife resampling** (see [http://en.wikipedia.org/wiki/Jackknife\\_resampling](http://en.wikipedia.org/wiki/Jackknife_resampling)). The idea behind jackknife resampling is to systematically generate datasets from the original dataset by leaving one value out at a time. In effect, we are trying to establish what will happen if at least one of the values is wrong. For each new generated dataset, we recalculate the arithmetic mean, variance, and standard deviation. This gives us an idea of how much those values can vary.

## Time for action – handling NaNs with the `nanmean()`, `nanvar()`, and `nanstd()` functions

We will apply jackknife resampling to the stock data. Each value will be omitted by setting it to **Not a Number (NaN)**. The `nanmean()`, `nanvar()`, and `nanstd()` can then be used to compute the arithmetic mean, variance, and standard deviation.

1. First, initialize a 30-by-3 array for the estimates as follows:
2. Loop through the values and generate a new dataset by setting one value to NaN at each iteration of the loop. For each new set of values, compute the estimates:

```
estimates = np.zeros((len(c), 3))

for i in xrange(len(c)):
    a = c.copy()
    a[i] = np.nan

    estimates[i,] = [np.nanmean(a), np.nanvar(a), np.nanstd(a)]
```

3. Print the variance for each estimate (you can also print the mean or standard deviation if you prefer):

```
print("Estimates variance", estimates.var(axis=0))
```

The following is printed on the screen:

```
Estimates variance [ 0.05960347  3.63062943  0.01868965]
```

### What just happened?

We estimated the variances of the arithmetic mean, variance, and standard deviation of a small dataset using jackknife resampling. This gives us an idea of how much the arithmetic mean, variance, and standard deviation vary. The code for this example can be found in the `jackknife.py` file in this book's code bundle:

```
from __future__ import print_function
import numpy as np

c = np.loadtxt('data.csv', delimiter=',', usecols=(6,), unpack=True)

# Initialize estimates array
estimates = np.zeros((len(c), 3))

for i in xrange(len(c)):
```

```
# Create a temporary copy and omit one value
a = c.copy()
a[i] = np.nan

# Compute estimates
estimates[i,] = [np.nanmean(a), np.nanvar(a), np.nanstd(a)]

print("Estimates variance", estimates.var(axis=0))
```

## Summary

This chapter informed us about a great number of common NumPy functions. A few common statistics functions were also mentioned.

After this tour through the common NumPy functions, we will continue covering convenience NumPy functions such as `polyfit()`, `sign()`, and `piecewise()` in the next chapter.

# 4

## Convenience Functions for Your Convenience

*As we have seen, NumPy has a great number of functions. Many of those functions exist just for convenience, and knowing them will greatly increase your productivity. This includes functions that select certain parts of your arrays (based on a Boolean condition, for instance) or manipulate polynomials. This chapter has an example of computing correlation to give you a taste of data analysis with NumPy.*

In this chapter, we shall cover the following topics:

- ◆ Data selection and extraction
- ◆ Simple data analysis
- ◆ Examples of correlation of returns
- ◆ Polynomials
- ◆ Linear algebra functions

In *Chapter 3, Getting Familiar with Commonly Used Functions*, we had one data file to play around with. Things have improved in this chapter—we now have two data files. Let's explore the data with NumPy.

## Correlation

Have you noticed that the stock price of some companies will be closely followed by another, usually a rival in the same sector? The theoretical explanation is that because these two companies are in the same type of business, they share the same challenges, require the same materials and resources, and compete for the same type of customers.

You could think of many possible pairs, but you need to check for a real relationship. One way is to take a look at the correlation of the stock returns of both stocks (see <https://www.khanacademy.org/math/probability/statistical-studies/types-of-studies/v/correlation-and-causality>). A high correlation implies a relationship of some sort. It is not proof of causality though, especially if you don't use sufficient data.

### Time for action – trading correlated pairs

For this section, we will use two sample datasets, containing end-of-day price data. The first company is BHP Billiton (BHP), which is active in mining of petroleum, metals, and diamonds. The second is Vale (VALE), which is also a metals and mining company. So, there is some overlap of activity, albeit not 100 percent. For evaluating correlated pairs, follow these steps:

1. First, load the data, specifically the close price of the two securities, from the CSV files in the example code directory of this chapter and calculate the returns. If you don't remember how to do it, look at the examples in *Chapter 3, Getting Familiar with Commonly Used Functions*.
2. Covariance tells us how two variables vary together; which is nothing more than unnormalized correlation (see <https://www.khanacademy.org/math/probability/regression/regression-correlation/v/covariance-and-the-regression-line>):

$$\text{cov}(a, b) = \frac{1}{N} \sum_{i=1}^N (a_i - \text{mean}(a))(b_i - \text{mean}(b))$$

Compute the covariance matrix from the returns with the `cov()` function (it's not strictly necessary to do this, but it will allow us to demonstrate a few matrix operations):

```
covariance = np.cov(bhp_returns, vale_returns)
print("Covariance", covariance)
```

The covariance matrix is as follows:

```
Covariance [[ 0.00028179  0.00019766]
            [ 0.00019766  0.00030123]]
```

- 3.** View the values on the diagonal with the `diagonal()` method:

```
print("Covariance diagonal", covariance.diagonal())
```

The diagonal values of the covariance matrix are as follows:

```
Covariance diagonal [ 0.00028179  0.00030123]
```

Notice that the values on the diagonal are not equal to each other. This is different from the correlation matrix.

- 4.** Compute the trace, the sum of the diagonal values, with the `trace()` method:

```
print("Covariance trace", covariance.trace())
```

The trace values of the covariance matrix are as follows:

```
Covariance trace 0.00058302354992
```

- 5.** The correlation of two vectors is defined as the covariance, divided by the product of the respective standard deviations of the vectors. The equation for vectors *a* and *b* is as follows:

```
print(covariance/ (bhp_returns.std() * vale_returns.std()))
```

The correlation matrix is as follows:

```
[[ 1.00173366  0.70264666]
 [ 0.70264666  1.0708476 ]]
```

- 6.** We will measure the correlation of our pair with the correlation coefficient. The correlation coefficient takes values between  $-1$  and  $1$ . The correlation of a set of values with itself is  $1$  by definition. This would be the ideal value; however, we will also be happy with a slightly lower value. Calculate the correlation coefficient (or, more accurately, the correlation matrix) with the `corrcoef()` function:

```
print("Correlation coefficient", np.corrcoef(bhp_returns, vale_returns))
```

The coefficients are as follows:

```
[[ 1.          0.67841747]
 [ 0.67841747  1.          ]]
```

The values on the diagonal are just the correlations of the BHP and VALE with themselves and are, therefore, equal to  $1$ . In all likelihood, no real calculation takes place. The other two values are equal to each other since correlation is symmetrical, meaning that the correlation of BHP with VALE is equal to the correlation of VALE with BHP. It seems that here the correlation is not that strong.



- 7.** Another important point is whether the two stocks under consideration are in sync or not. Two stocks are considered out of sync if their difference is two standard deviations from the mean of the differences.

If they are out of sync, we could initiate a trade, hoping that they will eventually get back in sync again. Compute the difference between the close prices of the two securities to check the synchronization:

```
difference = bhp - vale
```

Check whether the last difference in price is out of sync; see the following code:

```
avg = np.mean(difference)
dev = np.std(difference)
print("Out of sync", np.abs(difference[-1] - avg) > 2 * dev)
```

Unfortunately, we cannot trade yet:

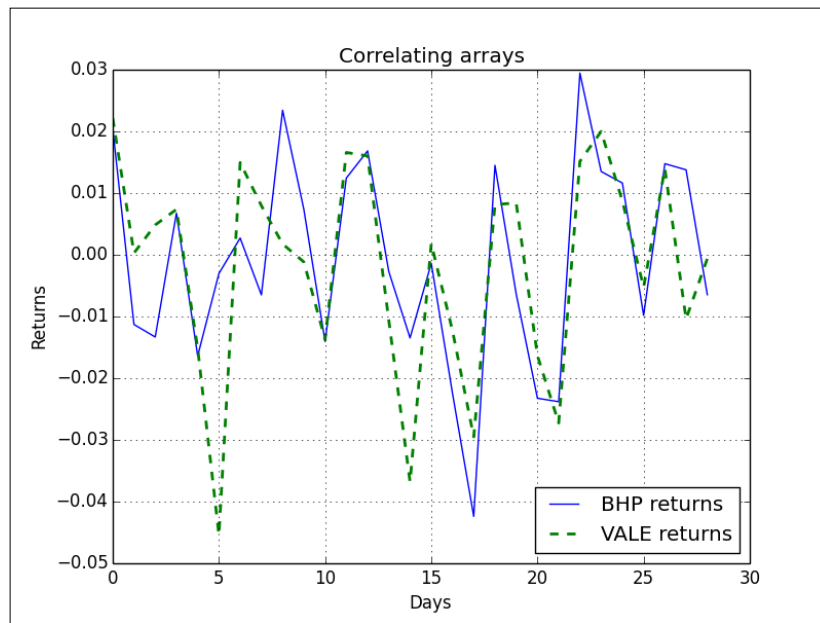
```
Out of sync False
```

- 8.** Plotting requires `matplotlib`; this will be discussed in *Chapter 9, Plotting with matplotlib*. Plotting can be done as follows:

```
t = np.arange(len(bhp_returns))
plt.plot(t, bhp_returns, lw=1, label='BHP returns')
plt.plot(t, vale_returns, '--', lw=2, label='VALE returns')
plt.title('Correlating arrays')

plt.xlabel('Days')
plt.ylabel('Returns')
plt.grid()
plt.legend(loc='best')
plt.show()
```

The resulting plot is shown here:



### ***What just happened?***

We analyzed the relation of the closing stock prices of BHP and VALE. To be precise, we calculated the correlation of their stock returns. We achieved this with the `corrcoef()` function. Furthermore, we saw how to compute the covariance matrix from which the correlation can be derived. As a bonus, we demonstrated the `diagonal()` and `trace()` methods that give us the diagonal values and the trace of a matrix, respectively. For the source code, see the `correlation.py` file in this book's code bundle:

```
from __future__ import print_function
import numpy as np
import matplotlib.pyplot as plt

bhp = np.loadtxt('BHP.csv', delimiter=',', usecols=(6,), unpack=True)

bhp_returns = np.diff(bhp) / bhp[:, -1]

vale = np.loadtxt('VALE.csv', delimiter=',', usecols=(6,),
unpack=True)

vale_returns = np.diff(vale) / vale[:, -1]

covariance = np.cov(bhp_returns, vale_returns)
```

```
print("Covariance", covariance)

print("Covariance diagonal", covariance.diagonal())
print("Covariance trace", covariance.trace())

print(covariance/ (bhp_returns.std() * vale_returns.std()))

print("Correlation coefficient", np.corrcoef(bhp_returns, vale_
returns))

difference = bhp - vale
avg = np.mean(difference)
dev = np.std(difference)

print("Out of sync", np.abs(difference[-1] - avg) > 2 * dev)

t = np.arange(len(bhp_returns))
plt.plot(t, bhp_returns, lw=1, label='BHP returns')
plt.plot(t, vale_returns, '--', lw=2, label='VALE returns')
plt.title('Correlating arrays')
plt.xlabel('Days')
plt.ylabel('Returns')
plt.grid()
plt.legend(loc='best')
plt.show()
```

## Pop quiz – calculating covariance

Q1. Which function returns the covariance of two arrays?

1. covariance
2. covar
3. cov
4. cvar

## Polynomials

Do you like calculus? Well I love it! One of the ideas in calculus is **Taylor expansion**, that is, representing a differentiable function as an infinite series (see [https://www.khanacademy.org/math/integral-calculus/sequences\\_series\\_approx\\_calc/taylor-series/v/generalized-taylor-series-approximation](https://www.khanacademy.org/math/integral-calculus/sequences_series_approx_calc/taylor-series/v/generalized-taylor-series-approximation) and [http://en.wikipedia.org/wiki/Taylor\\_series](http://en.wikipedia.org/wiki/Taylor_series)).

The Taylor series is defined as the following sum:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$



in this definition is the  $n$ th derivative of the function  $f$  computed at the point  $a$ .

In practice, this means that we can estimate any differentiable, and therefore continuous, function with a polynomial of a high degree. We would then assume that the terms of the higher degrees are negligibly small.

## Time for action – fitting to polynomials

The NumPy `polyfit()` function fits a set of data points to a polynomial, even if the underlying function is not continuous:

1. Continuing with the price data of BHP and VALE, look at the difference of their close prices and fit it to a polynomial of the third power:

```
bhp=np.loadtxt('BHP.csv', delimiter=',', usecols=(6,),
              unpack=True)
vale=np.loadtxt('VALE.csv', delimiter=',', usecols=(6,),
              unpack=True)
t = np.arange(len(bhp))
poly = np.polyfit(t, bhp - vale, 3)
print("Polynomial fit", poly)
```

The polynomial fit (in this example, a cubic polynomial was chosen) is as follows:

```
Polynomial fit [ 1.11655581e-03 -5.28581762e-02  5.80684638e-01
 5.79791202e+01]
```

2. The numbers you see are the coefficients of the polynomial. Extrapolate to the next value with the `polyval()` function and the polynomial object that we got from the fit:

```
print("Next value", np.polyval(poly, t[-1] + 1))
```

The next value we predict will be this:

```
Next value 57.9743076081
```

- 3.** Ideally, the difference between the close prices of BHP and VALE should be as small as possible. In an extreme case, it might be zero at some point. Find out when our polynomial fit reaches zero with the `roots()` function:

```
print("Roots", np.roots(poly))
```

The roots of the polynomial are as follows:

```
Roots [ 35.48624287+30.62717062j  35.48624287-30.62717062j
 -23.63210575 +0.j              ]
```

- 4.** Another thing you may have learned in calculus class was to find **extrema**—these could be potential maxima or minima. Remember, from calculus, that these are the points where the derivative of our function is zero. Differentiate the polynomial fit with the `polyder()` function:

```
der = np.polyder(poly)
print("Derivative", der)
```

The coefficients of the derivative polynomial are as follows:

```
Derivative [ 0.00334967 -0.10571635  0.58068464]
```

- 5.** Get the roots of the derivative:

```
print("Extremas", np.roots(der))
```

The extremas that we get are as follows:

```
Extremas [ 24.47820054  7.08205278]
```

Let's double-check and compute the values of the fit with the `polyval()` function:

```
vals = np.polyval(poly, t)
```

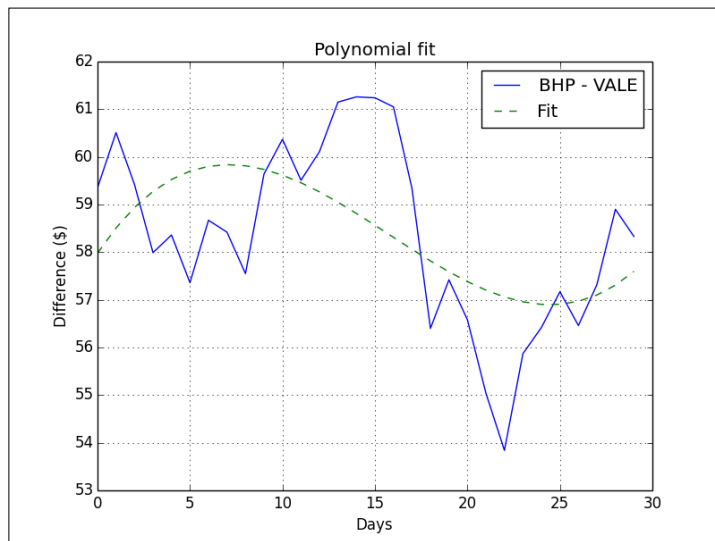
- 6.** Now, find the maximum and minimum values with the `argmax()` and the `argmin()` function:

```
vals = np.polyval(poly, t)
print(np.argmax(vals))
print(np.argmin(vals))
```

This gives us the expected results shown in the following screenshot. OK, not quite the same results, but, if we backtrack to step 1, we can see that `t` was defined with the `arange()` function:

```
7
24
```

Plot the data and the fit it to get the following plot:



Obviously, the smooth line is the fit and the jagged line is the underlying data. But as it's not that good a fit, you might want to try a higher order polynomial.

### ***What just happened?***

We fit data to a polynomial with the `polyfit()` function. We learned about the `polyval()` function that computes the values of a polynomial, the `roots()` function that returns the roots of the polynomial, and the `polyder()` function that gives back the derivative of a polynomial (see `polynomials.py`):

```
from __future__ import print_function
import numpy as np
import sys
import matplotlib.pyplot as plt

bhp=np.loadtxt('BHP.csv', delimiter=',', usecols=(6,), unpack=True)
vale=np.loadtxt('VALE.csv', delimiter=',', usecols=(6,), unpack=True)

t = np.arange(len(bhp))
poly = np.polyfit(t, bhp - vale, 3)
print("Polynomial fit", poly)

print("Next value", np.polyval(poly, t[-1] + 1))
```

```
print("Roots", np.roots(poly))

der = np.polyder(poly)
print("Derivative", der)

print("Extremas", np.roots(der))
vals = np.polyval(poly, t)
print(np.argmax(vals))
print(np.argmin(vals))

plt.plot(t, bhp - vale, label='BHP - VALE')
plt.plot(t, vals, '--', label='Fit')
plt.title('Polynomial fit')
plt.xlabel('Days')
plt.ylabel('Difference ($)')
plt.grid()
plt.legend()
plt.show()
```

## Have a go hero – improving the fit

You could do a number of things to improve the fit. For example, try a different power as, in this section, a cubic polynomial was chosen. Consider smoothing the data before fitting it. One way you could smooth the data is with a moving average. You can find examples of simple and EMA calculations in the *Chapter 3, Getting Familiar with Commonly Used Functions*.

## On-balance volume

Volume is a very important variable in investing; it indicates how big a price move is. The on-balance volume indicator is one of the simplest stock price indicators. It is based on the close price of the current and previous days and the volume of the current day. For each day, if the close price today is higher than the close price of yesterday, then the value of the on-balance volume is equal to the volume of today. On the other hand, if today's close price is lower than yesterday's close price, then the value of the on-balance volume indicator is the difference between the on-balance volume and the volume of today. However, if the close price did not change, then the value of the on-balance volume is zero.

## Time for action – balancing volume

In other words, we need to multiply the sign of the close price and the volume. In this section, we look at two approaches to this problem: one using the NumPy `sign()` function and the other using the NumPy `piecewise()` function.

1. Load the BHP data into a close and volume array:

```
c, v=np.loadtxt('BHP.csv', delimiter=',', usecols=(6, 7),
               unpack=True)
```

Compute the absolute value changes. Calculate the change of the closing price with the `diff()` function. The `diff()` function computes the difference between two sequential array elements and returns an array containing these differences:

```
change = np.diff(c)
print("Change", change)
```

The changes of the close price are shown as follows:

```
Change [ 1.92 -1.08 -1.26  0.63 -1.54 -0.28  0.25 -0.6   2.15
 0.69 -1.33  1.16
 1.59 -0.26 -1.29 -0.13 -2.12 -3.91  1.28 -0.57 -2.07 -2.07  2.5
 1.18
-0.88  1.31  1.24 -0.59]
```

2. The NumPy `sign()` function returns the signs for each element in an array. -1 is returned for a negative number, 1 for a positive number, and 0, otherwise. Apply the `sign()` function to the `change` array:

```
signs = np.sign(change)
print("Signs", signs)
```

The signs of the change array are as follows:

```
Signs [ 1. -1. -1.  1. -1. -1.  1. -1.  1.  1. -1.  1.  1. -1. -1.
-1. -1. -1.
-1. -1. -1.  1.  1.  1. -1.  1.  1. -1.]
```

Alternatively, we can calculate the signs with the `piecewise()` function. The `piecewise()` function, as its name suggests, evaluates a function piece-by-piece. Call the function with the appropriate return values and conditions:

```
pieces = np.piecewise(change, [change < 0, change > 0], [-1,
 1])
print("Pieces", pieces)
```



The signs are shown again as follows:

```
Pieces [ 1. -1. -1.  1. -1. -1.  1. -1.  1.  1. -1.  1.  1. -1.
-1. -1. -1. -1.
-1. -1. -1.  1.  1.  1. -1.  1.  1. -1.]
```

Check that the outcome is the same:

```
print("Arrays equal?", np.array_equal(signs, pieces))
```

And the outcome is as follows:

```
Arrays equal? True
```

3. The on-balance volume depends on the change of the previous close, so we cannot calculate it for the first day in our sample:

```
print("On balance volume", v[1:] * signs)
```

The on-balance volume is as follows:

```
[ 2620800. -2461300. -3270900.  2650200. -4667300. -5359800.
 7768400.
 -4799100.  3448300.  4719800. -3898900.  3727700.  3379400.
-2463900.
 -3590900. -3805000. -3271700. -5507800.  2996800. -3434800.
-5008300.
 -7809799.  3947100.  3809700.  3098200. -3500200.  4285600.
 3918800.
 -3632200.]
```

## ***What just happened?***

We computed the on-balance volume that depends on the change of the closing price. Using the NumPy `sign()` and `piecewise()` functions, we went over two different methods to determine the sign of the change (see `obv.py`) as follows:

```
from __future__ import print_function
import numpy as np

c, v=np.loadtxt('BHP.csv', delimiter=',', usecols=(6, 7), unpack=True)

change = np.diff(c)
print("Change", change)

signs = np.sign(change)
print("Signs", signs)
```

```

pieces = np.piecewise(change, [change < 0, change > 0], [-1, 1])
print("Pieces", pieces)

print("Arrays equal?", np.array_equal(signs, pieces))

print("On balance volume", v[1:] * signs)

```

## Simulation

Often, you would want to try something out first. Play around, experiment, but preferably without blowing things up or getting dirty! NumPy is perfect for experimentation. We will use NumPy to simulate a trading day, without actually losing money. Many people like to buy on the dip or, in other words, wait for the price of stocks to drop before buying. A variant of this is to wait for the price to drop a small percentage, say 0.1 percent below the opening price of the day.

### Time for action – avoiding loops with `vectorize()`

The `vectorize()` function is a yet another trick to reduce the number of loops in your programs. Calculate the profit of a single trading day following these steps:

1. First, load the data:
 

```
o, h, l, c = np.loadtxt('BHP.csv', delimiter=',', usecols=(3, 4, 5, 6), unpack=True)
```
2. The `vectorize()` function is the NumPy equivalent of the Python `map()` function. Call the `vectorize()` function, giving it as an argument the `calc_profit()` function:
 

```
func = np.vectorize(calc_profit)
```
3. We can now apply `func()` as if it is a function. Apply the `func()` function result that we got to the price arrays:
 

```
profits = func(o, h, l, c)
```
4. The `calc_profit()` function is pretty simple. First, we try to buy slightly below the open price. If this is outside of the daily range, then, obviously, our attempt failed and no profit was made, or we incurred a loss, therefore, will return 0. Otherwise, we sell at the close price and the profit is simply the difference between the buy price and the close price. Actually, it is, in fact, more interesting to have a look at the relative profit:

```

def calc_profit(open, high, low, close):
    #buy just below the open
    buy = open * 0.999

```

```
# daily range
if low < buy < high:
    return (close - buy)/buy
else:
    return 0
```

```
print("Profits", profits)
```

5. Assume that there are two days with zero profits, where there was either no net gain or a loss. Select the days with trades and calculate the averages:

```
real_trades = profits[profits != 0]
print("Number of trades", len(real_trades), round(100.0 *
    len(real_trades)/len(c), 2), "%")
print("Average profit/loss %", round(np.mean(real_trades) * 100,
2))
```

The trades summary is shown as follows:

```
Number of trades 28 93.33 %
Average profit/loss % 0.02
```

6. As optimists, we are interested in winning trades with a gain greater than zero. Select the days with winning trades and calculate the averages:

```
winning_trades = profits[profits > 0]
print("Number of winning trades", len(winning_trades),
    round(100.0 * len(winning_trades)/len(c), 2), "%")
print("Average profit %", round(np.mean(winning_trades) * 100,
2))
```

The winning trades statistics are as follows:

```
Number of winning trades 16 53.33 %
Average profit % 0.72
```

7. Alternatively, as pessimists, we are interested in losing trades with a profit less than zero. Select the days with losing trades and calculate the averages:

```
losing_trades = profits[profits < 0]
print("Number of losing trades", len(losing_trades), round(100.0 *
    len(losing_trades)/len(c), 2), "%")
print("Average loss %", round(np.mean(losing_trades) * 100, 2))
```

The losing trades statistics are as follows:

```
Number of losing trades 12 40.0 %
Average loss % -0.92
```

## ***What just happened?***

We vectorized a function, which is just another way to avoid using loops. We simulated a trading day with a function, which returned the relative profit of each day's trade. We printed a statistics summary of the losing and winning trades (see `simulation.py`):

```
from __future__ import print_function
import numpy as np

o, h, l, c = np.loadtxt('BHP.csv', delimiter=',', usecols=(3, 4, 5,
6), unpack=True)

def calc_profit(open, high, low, close):
    #buy just below the open
    buy = open * 0.999

    # daily range
    if low < buy < high:
        return (close - buy)/buy
    else:
        return 0

func = np.vectorize(calc_profit)
profits = func(o, h, l, c)
print("Profits", profits)

real_trades = profits[profits != 0]
print("Number of trades", len(real_trades), round(100.0 * len(real_
trades)/len(c), 2), "%")
print("Average profit/loss %", round(np.mean(real_trades) * 100, 2))

winning_trades = profits[profits > 0]
print("Number of winning trades", len(winning_trades), round(100.0 *
len(winning_trades)/len(c), 2), "%")
print("Average profit %", round(np.mean(winning_trades) * 100, 2))

losing_trades = profits[profits < 0]
print("Number of losing trades", len(losing_trades), round(100.0 *
len(losing_trades)/len(c), 2), "%")
print("Average loss %", round(np.mean(losing_trades) * 100, 2))
```

## Have a go hero – analyzing consecutive wins and losses

Although the average profit is positive, it is also important to know whether we had to endure a long streak of consecutive losses. If this is the case, we might be left with little or no capital, and then the average profit would not matter.

Find out if there was such a losing streak. If you want, you can also find out if there was a prolonged winning streak.

## Smoothing

Noisy data is difficult to deal with, so we often need to do some smoothing. Besides calculating moving averages, we can use one of the NumPy functions to smooth data.

The `hanning()` function is a window function formed by a weighted cosine (see [http://en.wikipedia.org/wiki/Hann\\_function](http://en.wikipedia.org/wiki/Hann_function)):

$$w(n) = 0.5 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right)$$

In the preceding formula,  $N$  corresponds to the size of the window. We will cover the other window functions in later chapters.

## Time for action – smoothing with the `hanning()` function

We will use the `hanning()` function to smooth arrays of stock returns, as shown in the following steps:

1. Call the `hanning()` function to compute weights for a certain length window (in this example 8) as follows:

```
N = 8
weights = np.hanning(N)
print("Weights", weights)
```

The weights are as follows:

```
Weights [ 0.          0.1882551  0.61126047  0.95048443
 0.95048443  0.61126047
 0.1882551  0.          ]
```

2. Calculate the stock returns for the BHP and VALE quotes using `convolve()` with normalized weights:

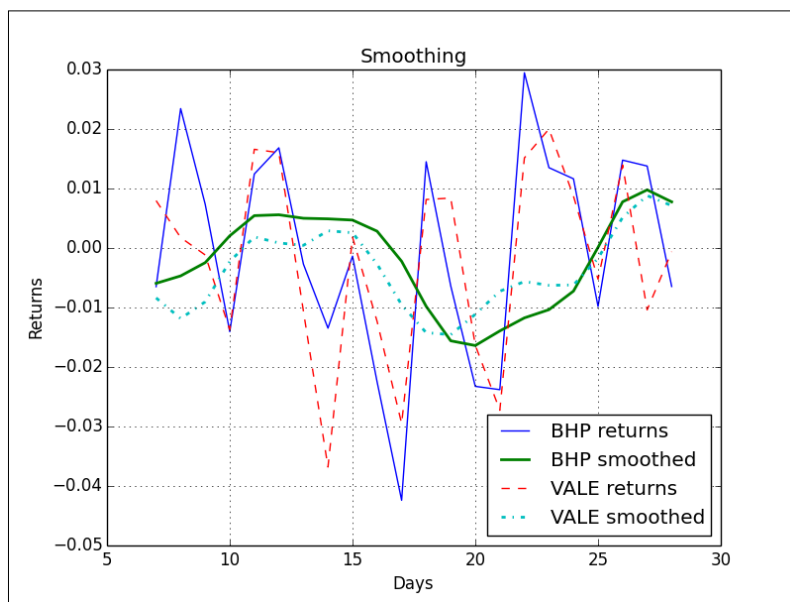
```
bhp = np.loadtxt('BHP.csv', delimiter=',', usecols=(6,),
unpack=True)
bhp_returns = np.diff(bhp) / bhp[ : -1]
smooth_bhp = np.convolve(weights/weights.sum(),
    bhp_returns)[N-1:-N+1]

vale = np.loadtxt('VALE.csv', delimiter=',', usecols=(6,),
unpack=True)
vale_returns = np.diff(vale) / vale[ : -1]
smooth_vale = np.convolve(weights/weights.sum(),
    vale_returns)[N-1:-N+1]
```

3. Plot with `matplotlib` using this code:

```
t = np.arange(N - 1, len(bhp_returns))
plt.plot(t, bhp_returns[N-1:], lw=1.0)
plt.plot(t, smooth_bhp, lw=2.0)
plt.plot(t, vale_returns[N-1:], lw=1.0)
plt.plot(t, smooth_vale, lw=2.0)
plt.show()
```

The chart would appear as follows:



The thin lines on the preceding chart are the stock returns and the thick lines are the result of smoothing. As you can see, the lines cross a few times. These points might be important because the trend might have changed there. Or, at least, the relation of BHP to VALE might have changed. These turning inflection points probably occur often, so we might want to project into the future.

4. Fit the result of the smoothing step to polynomials as follows:

```
K = 8
t = np.arange(N - 1, len(bhp_returns))
poly_bhp = np.polyfit(t, smooth_bhp, K)
poly_vale = np.polyfit(t, smooth_vale, K)
```

5. Next, we need to evaluate the situation, where the polynomials we found in the previous step were equal to each other. This boils down to subtracting the polynomials and finding the roots of the resulting polynomial. Subtract the polynomials using `polysub()`:

```
poly_sub = np.polysub(poly_bhp, poly_vale)
xpoints = np.roots(poly_sub)
print("Intersection points", xpoints)
```

The points are shown as follows:

```
Intersection points [ 27.73321597+0.j          27.51284094+0.j
 24.32064343+0.j
 18.86423973+0.j          12.43797190+1.73218179j  12.43797190-
 1.73218179j
 6.34613053+0.62519463j   6.34613053-0.62519463j]
```

6. The numbers we get are complex, and that is not good for us (unless there is such a thing as imaginary time). Check which numbers are real with the `isreal()` function:

```
reals = np.isreal(xpoints)
print("Real number?", reals)
```

The result is as follows:

```
Real number? [ True  True  True  True False False False False]
```

Some of the numbers are real, so select them with the `select()` function. The `select()` function forms an array by taking elements from a list of choices, based on a list of conditions:

```
xpoints = np.select([reals], [xpoints])
xpoints = xpoints.real
print("Real intersection points", xpoints)
```

The real intersection points are as follows:

```
Real intersection points [ 27.73321597  27.51284094  24.32064343
 18.86423973  0.          0.  0.  0.]
```

7. We managed to pick up some zeroes. The `trim_zeros()` function strips the leading and trailing zeroes from a one-dimensional array. Get rid of the zeroes with the `trim_zeros()` function:

```
print("Sans 0s", np.trim_zeros(xpoints))
```

The zeroes are gone, and the output is shown as follows:

```
Sans 0s [ 27.73321597  27.51284094  24.32064343  18.86423973]
```

## ***What just happened?***

We applied the `hanning()` function to smooth arrays containing stock returns. We subtracted two polynomials with the `polysub()` function. We then checked for real numbers with the `isreal()` function and selected the real numbers with the `select()` function. Finally, we stripped zeroes from an array with the `trim_zeros()` function (see `smoothing.py`):

```
from __future__ import print_function
import numpy as np
import matplotlib.pyplot as plt

N = 8

weights = np.hanning(N)
print("Weights", weights)

bhp = np.loadtxt('BHP.csv', delimiter=',', usecols=(6,), unpack=True)
bhp_returns = np.diff(bhp) / bhp[:, -1]
smooth_bhp = np.convolve(weights/weights.sum(), bhp_returns)[N-1:-N+1]

vale = np.loadtxt('VALE.csv', delimiter=',', usecols=(6,),
unpack=True)
vale_returns = np.diff(vale) / vale[:, -1]
smooth_vale = np.convolve(weights/weights.sum(), vale_returns)
[N-1:-N+1]

K = 8
t = np.arange(N - 1, len(bhp_returns))
poly_bhp = np.polyfit(t, smooth_bhp, K)
```



```
poly_vale = np.polyfit(t, smooth_vale, K)

poly_sub = np.polysub(poly_bhp, poly_vale)
xpoints = np.roots(poly_sub)
print("Intersection points", xpoints)

reals = np.isreal(xpoints)
print("Real number?", reals)

xpoints = np.select([reals], [xpoints])
xpoints = xpoints.real
print("Real intersection points", xpoints)

print("Sans 0s", np.trim_zeros(xpoints))

plt.plot(t, bhp_returns[N-1:], lw=1.0, label='BHP returns')
plt.plot(t, smooth_bhp, lw=2.0, label='BHP smoothed')

plt.plot(t, vale_returns[N-1:], '--', lw=1.0, label='VALE returns')
plt.plot(t, smooth_vale, '-.', lw=2.0, label='VALE smoothed')
plt.title('Smoothing')
plt.xlabel('Days')
plt.ylabel('Returns')
plt.grid()
plt.legend(loc='best')
plt.show()
```

## Have a go hero – smoothing variations

Experiment with the other smoothing functions—`hamming()`, `blackman()`, `bartlett()`, and `kaiser()`. They work in more or less the same way as the `hanning()` function.

## Initialization

So far in this book, we encountered several convenient functions for initializing arrays. The `full()` and `full_like()` functions were recently added to NumPy to make initialization even easier.

The following short Python session shows (abbreviated) documentation for these two functions:

```
$ python
>>> import numpy as np
>>> help(np.full)
Return a new array of given shape and type, filled with `fill_value`.
>>> help(np.full_like)
```

Return a full array with the same shape and type as a given array.

## Time for action – creating value initialized arrays with the `full()` and `full_like()` functions

Let's demonstrate how the `full()` and `full_like()` functions work. If you are not in a Python shell already, type the following:

```
$ python
>>> import numpy as np
```

1. Create a one-by-two array with the `full()` function filled with the number 42 as follows:

```
>>> np.full((1, 2), 42)
array([[ 42.,  42.]])
```

As you can deduce from the output, the array elements are floating-point numbers, which is the default data type for NumPy arrays. Specify an integer data type as follows:

```
>>> np.full((1, 2), 42, dtype=np.int)
array([[42, 42]])
```

2. The `full_like()` function looks at the metadata of an input array and uses that information to create a new array, filled with a specified value. For instance, after creating an array with the `linspace()` function, use that as a template for the `full_like()` function:

```
>>> a = np.linspace(0, 1, 5)
>>> a
array([ 0. ,  0.25,  0.5 ,  0.75,  1. ])
>>> np.full_like(a, 42)
array([ 42.,  42.,  42.,  42.,  42.])
```

Again we have an array filled with 42. To change the data type to integer, type the following:

```
>>> np.full_like(a, 42, dtype=np.int)
array([42, 42, 42, 42, 42])
```

## ***What just happened?***

We created arrays using the `full()` and `full_like()` functions. The `full()` function filled the array with the number 42. The `full_like()` function uses the metadata of an input array to create a new array. Both functions allow you to specify the data type.

## **Summary**

We calculated the correlation of the stock returns of two stocks with the `corrcoef()` function. As a bonus, we demonstrated the `diagonal()` and `trace()` functions, which can give us the diagonal and trace of a matrix.

We fit data to a polynomial with the `polyfit()` function. We learned about the `polyval()` function that computes the values of a polynomial, the `roots()` function that returns the roots of the polynomial, and the `polyder()` function, which gives back the derivative of a polynomial.

We saw that the `full()` function fills an array with a number, and the `full_like()` function uses the metadata of an input array to create a new array. Both functions allow you to specify the data type.

Hopefully, you have increased your productivity, so that we can continue in the next chapter with matrices and **Universal Functions (ufuncs)**.

# 5

## Working with Matrices and ufuncs

*This chapter covers matrices and Universal functions (ufuncs). Matrices are well known in mathematics and have their representation in NumPy as well. Universal functions work on arrays, element by element, or on scalars. ufuncs expect a set of scalars as input and produce a set of scalars as output. Universal functions can typically be mapped to their mathematical counterparts such as add, subtract, divide, multiply, and so on. We will also introduce trigonometric, bitwise, and comparison universal functions.*

In this chapter, we will cover the following topics:

- ◆ Matrix creation
- ◆ Matrix operations
- ◆ Basic ufuncs
- ◆ Trigonometric functions
- ◆ Bitwise functions
- ◆ Comparison functions

## Matrices

Matrices in NumPy are subclasses of `ndarray`. We can create matrices using a special string format. They are, just like in mathematics, two-dimensional (see <https://www.khanacademy.org/math/precalculus/precalc-matrices>). Matrix multiplication is, as you would expect, different from the normal NumPy multiplication. The same is true for the power operator. We can create matrices with the `mat()`, `matrix()`, and `bmat()` functions.

### Time for action – creating matrices

The `mat()` function does not make a copy if the input is already a matrix or an `ndarray`. Calling this function is equivalent to calling `matrix(data, copy=False)`. We will also demonstrate transposing and inverting matrices.

1. Rows are delimited by a semicolon and values by a space. Call the `mat()` function with the following string to create a matrix:

```
A = np.mat('1 2 3; 4 5 6; 7 8 9')
print("Creation from string", A)
```

The matrix output should be the following matrix:

```
Creation from string [[1 2 3]
 [4 5 6]
 [7 8 9]]
```

2. Transpose the matrix with the `T` attribute as follows:

```
print("transpose A", A.T)
```

The following is the transposed matrix:

```
transpose A [[1 4 7]
 [2 5 8]
 [3 6 9]]
```

3. The matrix can be inverted with the `I` attribute as follows (see [https://www.khanacademy.org/math/precalculus/precalc-matrices/inverting\\_matrices/v/inverse-matrix-part-1](https://www.khanacademy.org/math/precalculus/precalc-matrices/inverting_matrices/v/inverse-matrix-part-1)):

```
print("Inverse A", A.I)
```

The inverse matrix is printed as follows (be warned that this is a  $O(n^3)$  operation, meaning that it takes on average cubic time):

```
Inverse A [[ -4.50359963e+15   9.00719925e+15  -4.50359963e+15]
 [  9.00719925e+15  -1.80143985e+16   9.00719925e+15]
 [ -4.50359963e+15   9.00719925e+15  -4.50359963e+15]]
```

4. Instead of using a string to create a matrix, do it with an array:

```
print("Creation from array", np.mat(np.arange(9).reshape(3, 3)))
```

The newly created array is printed as follows:

```
Creation from array [[0 1 2]
 [3 4 5]
 [6 7 8]]
```

### ***What just happened?***

We created matrices with the `mat()` function. We transposed the matrices with the `T` attribute and inverted them with the `I` attribute (see `matrixcreation.py`):

```
from __future__ import print_function
import numpy as np

A = np.mat('1 2 3; 4 5 6; 7 8 9')
print("Creation from string", A)
print("transpose A", A.T)
print("Inverse A", A.I)
print("Check Inverse", A * A.I)

print("Creation from array", np.mat(np.arange(9).reshape(3, 3)))
```

## **Creating a matrix from other matrices**

Sometimes, we want to create a matrix from other smaller matrices. We can do this with the `bmat()` function. The `b` here stands for block matrix.

### **Time for action – creating a matrix from other matrices**

We will create a matrix from two smaller matrices as follows:

1. First, create a 2-by-2 identity matrix:

```
A = np.eye(2)
print("A", A)
```

The identity matrix looks like the following:

```
A [[ 1.  0.]
 [ 0.  1.]]
```

2. Create another matrix like `A` and multiply it by 2:

```
B = 2 * A
print("B", B)
```

The second matrix is as follows:

```
B [[ 2.  0.]
   [ 0.  2.]]
```

3. Create the compound matrix from a string. The string uses the same format as the `mat()` function—use matrices instead of numbers:

```
print("Compound matrix\n", np.bmat("A B; A B"))
```

The compound matrix is shown as follows:

```
Compound matrix
[[ 1.  0.  2.  0.]
 [ 0.  1.  0.  2.]
 [ 1.  0.  2.  0.]
 [ 0.  1.  0.  2.]]
```

## What just happened?

We created a block matrix from two smaller matrices with the `bmat()` function. We gave the function a string containing the names of matrices instead of numbers (see `bmatcreation.py`):

```
from __future__ import print_function
import numpy as np

A = np.eye(2)
print("A", A)
B = 2 * A
print("B", B)
print("Compound matrix\n", np.bmat("A B; A B"))
```

## Pop quiz – defining a matrix with a string

Q1. What is the row delimiter in a string accepted by the `mat()` and `bmat()` functions?

1. Semicolon
2. Colon
3. Comma
4. Space

## Universal functions

**Universal functions (ufuncs)** expect a set of scalars as input and produce a set of scalars as output. They are actually Python objects that encapsulate the behavior of a function. We can typically map ufuncs to their mathematical counterparts such as add, subtract, divide, multiply, and so on. Universal functions are, in general, faster because of their special optimizations and because they run on the native level.

### Time for action – creating universal functions

We can create a ufunc from a Python function with the NumPy the `frompyfunc()` function as follows:

1. Define a Python function that answers the ultimate question to the universe, existence, and the rest (it's from *The Hitchhiker's Guide to the Galaxy*, Douglas Adams, Pan Books, if you haven't read it, you can safely ignore this!):

```
def ultimate_answer(a):
```

So far, nothing special; we gave the function the name `ultimate_answer()` and defined one parameter, `a`.

2. Create a result consisting of all zeros that has the same shape as `a`, with the `zeros_like()` function:
 

```
result = np.zeros_like(a)
```
3. Now, set the elements of the initialized array to the answer 42 and return the result. The complete function should appear as shown in the following code snippet. The `flat` attribute gives us access to a flat iterator that allows us to set the value of the array.

```
def ultimate_answer(a):
    result = np.zeros_like(a)
    result.flat = 42
    return result
```

4. Create a ufunc with `frompyfunc()`; specify 1 as the number of input parameter followed by 1 as the number of output parameters:

```
ufunc = np.frompyfunc(ultimate_answer, 1, 1)
print("The answer", ufunc(np.arange(4)))
```

The result for a one-dimensional array is shown as follows:

```
The answer [42 42 42 42]
```



Do the same for a two-dimensional array with the following code:

```
print("The answer", ufunc(np.arange(4).reshape(2, 2)))
```

The output for a two dimensional array is shown as follows:

```
The answer [[42 42]
            [42 42]]
```

## ***What just happened?***

We defined a Python function. In this function, we initialized to zero the elements of an array, based on the shape of an input argument, with the `zeros_like()` function. Then, with the `flat` attribute of `ndarray`, we set the array elements to the ultimate answer, 42 (see `answer42.py`):

```
from __future__ import print_function
import numpy as np

def ultimate_answer(a):
    result = np.zeros_like(a)
    result.flat = 42
    return result

ufunc = np.frompyfunc(ultimate_answer, 1, 1)
print("The answer", ufunc(np.arange(4)))

print("The answer", ufunc(np.arange(4).reshape(2, 2)))
```

## **Universal function methods**

How can functions have methods? As we said earlier, universal functions are not functions but Python objects representing functions. Universal functions have five important methods listed as follows:

1. `ufunc.reduce(a[, axis, dtype, out, keepdims])`
2. `ufunc.accumulate(array[, axis, dtype, out])`
3. `ufunc.reduceat(a, indices[, axis, dtype, out])`
4. `ufunc.outer(A, B)`
5. `ufunc.at(a, indices[, b])`

## Time for action – applying the ufunc methods to the add function

Let's call the first four methods on the `add()` function:

1. The universal function reduces the input array recursively along a specified axis on consecutive elements. For the `add()` function, the result of reducing is similar to calculating the sum of an array. Call the `reduce()` method:

```
a = np.arange(9)
print("Reduce", np.add.reduce(a))
```

The reduced array should be as follows:

```
Reduce 36
```

2. The `accumulate()` method also recursively goes through the input array. But, contrary to the `reduce()` method, it stores the intermediate results in an array and returns that. The result, in the case of the `add()` function, is equivalent to calling the `cumsum()` function. Call the `accumulate()` method on the `add()` function:

```
print("Accumulate", np.add.accumulate(a))
```

The accumulated array is as follows:

```
Accumulate [ 0  1  3  6 10 15 21 28 36]
```

3. The `reduceat()` method is a bit complicated to explain, so let's call it and go through its algorithm, step by step. The `reduceat()` method requires as arguments an input array and a list of indices:

```
print("Reduceat", np.add.reduceat(a, [0, 5, 2, 7]))
```

The result is shown as follows:

```
Reduceat [10  5 20 15]
```

The first step concerns the indices 0 and 5. This step results in a reduce operation of the array elements between indices 0 and 5:

```
print("Reduceat step I", np.add.reduce(a[0:5]))
```

The output of step 1 is as follows:

```
Reduceat step I 10
```

The second step concerns indices 5 and 2. Since 2 is less than 5, the array element at index 5 is returned:

```
print("Reduceat step II", a[5])
```

The second step results in the following output:

```
Reduceat step II 5
```

The third step concerns indices 2 and 7. This step results in a reduce operation of the array elements between indices 2 and 7:

```
print("Reduceat step III", np.add.reduce(a[2:7]))
```

The result of the third step is shown as follows:

```
Reduceat step III 20
```

The fourth step concerns index 7. This step results in a reduce operation of the array elements from index 7 to the end of the array:

```
print("Reduceat step IV", np.add.reduce(a[7:]))
```

The fourth step result is shown as follows:

```
Reduceat step IV 15
```

4. The `outer()` method returns an array that has a rank, which is the sum of the ranks of its two input arrays. The method is applied to all possible pairs of the input array elements. Call the `outer()` method on the `add()` function:

```
print("Outer", np.add.outer(np.arange(3), a))
```

The outer sum output result is as follows:

```
Outer [[ 0  1  2  3  4  5  6  7  8]
       [ 1  2  3  4  5  6  7  8  9]
       [ 2  3  4  5  6  7  8  9 10]]
```

## ***What just happened?***

We applied the first four methods, `reduce()`, `accumulate()`, `reduceat()`, and `outer()`, of universal functions to the `add()` function (see `ufuncmethods.py`):

```
from __future__ import print_function
import numpy as np

a = np.arange(9)

print("Reduce", np.add.reduce(a))
print("Accumulate", np.add.accumulate(a))
print("Reduceat", np.add.reduceat(a, [0, 5, 2, 7]))
print("Reduceat step I", np.add.reduce(a[0:5]))
print("Reduceat step II", a[5])
```

```
print("Reduceat step III", np.add.reduce(a[2:7]))
print("Reduceat step IV", np.add.reduce(a[7:]))
print("Outer", np.add.outer(np.arange(3), a))
```

## Arithmetic functions

The common arithmetic operators `+`, `-`, and `*` are implicitly linked to the `add`, `subtract`, and `multiply` universal functions, respectively. This means that when you use one of these operators on a NumPy array, the corresponding universal function will get called. Division involves a slightly more complex process. The three universal functions that have to do with array division are `divide()`, `true_divide()`, and `floor_division()`. Two operators correspond to division: `/` and `//`.

### Time for action – dividing arrays

Let's see the array division in action:

1. The `divide()` function does truncated integer division and normal floating-point division:

```
a = np.array([2, 6, 5])
b = np.array([1, 2, 3])
print("Divide", np.divide(a, b), np.divide(b, a))
```

The result of the `divide()` function is shown as follows:

```
Divide [2 3 1] [0 0 0]
```

As you can see, truncation took place.

2. The `true_divide()` function comes closer to the mathematical definition of division. Integer division returns a floating-point result and no truncation occurs:

```
print("True Divide", np.true_divide(a, b), np.true_divide(b, a))
```

The result of the `true_divide()` function is as follows:

```
True Divide [ 2.          3.          1.66666667] [ 0.5
0.33333333  0.6          ]
```

3. The `floor_divide()` function always returns an integer result. It is equivalent to calling the `floor()` function after calling the `divide()` function. The `floor()` function discards the decimal part of a floating-point number and returns an integer:

```
print("Floor Divide", np.floor_divide(a, b), np.floor_divide(b, a))
c = 3.14 * b
print("Floor Divide 2", np.floor_divide(c, b),
      np.floor_divide(b, c))
```

The `floor_divide()` function call results in:

```
Floor Divide [2 3 1] [0 0 0]
Floor Divide 2 [ 3.  3.  3.] [ 0.  0.  0.]
```

4. By default, the `/` operator is equivalent to calling the `divide()` function:

```
from __future__ import division
```

However, if this line is found at the beginning of a Python program, the `true_divide()` function is called instead. So, this code will appear as follows:

```
print("/ operator", a/b, b/a)
```

The result is shown as follows:

```
/ operator [ 2.          3.          1.66666667] [ 0.5
0.33333333  0.6          ]
```

5. The `//` operator is equivalent to calling the `floor_divide()` function. For example, look at the following code snippet:

```
print("// operator", a//b, b//a)
print("// operator 2", c//b, b//c)
```

The `//` operator result is shown as follows:

```
// operator [2 3 1] [0 0 0]
// operator 2 [ 3.  3.  3.] [ 0.  0.  0.]
```

## What just happened?

The `divide()` function truncates the integer division and normal floating-point division. The `true_divide()` function always returns a floating-point result without any truncation. The `floor_divide()` function always returns an integer result; the result is the same that you will get by calling the `divide()` and `floor()` functions consecutively (see `dividing.py`):

```
from __future__ import print_function
from __future__ import division
import numpy as np

a = np.array([2, 6, 5])
b = np.array([1, 2, 3])

print("Divide", np.divide(a, b), np.divide(b, a))
print("True Divide", np.true_divide(a, b), np.true_divide(b, a))
```

```

print("Floor Divide", np.floor_divide(a, b), np.floor_divide(b, a))
c = 3.14 * b
print("Floor Divide 2", np.floor_divide(c, b), np.floor_divide(b, c))
print("/ operator", a/b, b/a)
print("// operator", a//b, b//a)
print("// operator 2", c//b, b//c)

```

## Have a go hero – experimenting with `__future__.division`

Experiment to confirm the impact of importing `__future__.division`.

## Modulo operation

We can calculate the modulo or remainder using the NumPy `mod()`, `remainder()`, and `fmod()` functions. Also, we can use the `%` operator. The main difference among these functions is how they deal with negative numbers. The odd one out in this group is the `fmod()` function.

## Time for action – computing the modulo

Let's call the previously mentioned functions:

1. The `remainder()` function returns the remainder of the two arrays, element-wise. 0 is returned if the second number is 0:

```

a = np.arange(-4, 4)
print("Remainder", np.remainder(a, 2))

```

The result of the `remainder()` function is shown as follows:

```
Remainder [0 1 0 1 0 1 0 1]
```

2. The `mod()` function does exactly the same as the `remainder()` function:

```
print("Mod", np.mod(a, 2))
```

The result of the `mod()` function is shown as follows:

```
Mod [0 1 0 1 0 1 0 1]
```

3. The `%` operator is just shorthand for the `remainder()` function:

```
print("% operator", a % 2)
```

The result of the `%` operator is shown as follows:

```
% operator [0 1 0 1 0 1 0 1]
```

4. The `fmod()` function handles negative numbers differently than `mod()`, `fmod()`, and `%` do. The sign of the remainder is the sign of the dividend, and the sign of the divisor has no influence on the results:

```
print("Fmod", np.fmod(a, 2))
```

The `fmod()` result is printed as follows:

```
Fmod [ 0 -1  0 -1  0  1  0  1]
```

### **What just happened?**

We demonstrated the NumPy the `mod()`, `remainder()`, and `fmod()` functions, which compute the modulo or remainder (see `modulo.py`):

```
from __future__ import print_function
import numpy as np

a = np.arange(-4, 4)

print("Remainder", np.remainder(a, 2))
print("Mod", np.mod(a, 2))
print("% operator", a % 2)
print("Fmod", np.fmod(a, 2))
```

## **Fibonacci numbers**

The **Fibonacci numbers** (see [http://en.wikipedia.org/wiki/Fibonacci\\_number](http://en.wikipedia.org/wiki/Fibonacci_number)) are based on a recurrence relation:

$$F_n = F_{n-1} + F_{n-2}$$

It is difficult to express this relation directly with NumPy code. However, we can express this relation in a matrix form or use the following **golden ratio** formula:

$$F_n = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}}$$

with

$$\varphi = \frac{1 + \sqrt{5}}{2}$$

This will introduce the `matrix()` and `rint()` functions. The `matrix()` function creates matrices and the `rint()` function rounds numbers to the closest integer, but the result is not an integer.

## Time for action – computing Fibonacci numbers

A matrix can represent the Fibonacci recurrence relation. We can express the calculation of Fibonacci numbers as a repeated matrix multiplication:

1. Create the Fibonacci matrix as follows:

```
F = np.matrix([[1, 1], [1, 0]])
print("F", F)
```

The Fibonacci matrix appears as follows:

```
F [[1 1]
   [1 0]]
```

2. Calculate the 8th Fibonacci number (ignoring 0), by subtracting 1 from 8 and taking the power of the matrix. The Fibonacci number then appears on the diagonal:

```
print("8th Fibonacci", (F ** 7)[0, 0])
```

The Fibonacci number is as follows:

```
8th Fibonacci 21
```

3. The **golden ratio** formula, better known as **Binet's** formula, allows us to calculate Fibonacci numbers with a rounding step at the end. Calculate the first eight Fibonacci numbers:

```
n = np.arange(1, 9)
sqrt5 = np.sqrt(5)
phi = (1 + sqrt5)/2
fibonacci = np.rint((phi**n - (-1/phi)**n)/sqrt5)
print("Fibonacci", fibonacci)
```

The first eight Fibonacci numbers are as follows:

```
Fibonacci [ 1.  1.  2.  3.  5.  8. 13. 21.]
```



## What just happened?

We computed Fibonacci numbers in two ways. In the process, we learned about the `matrix()` function that creates matrices. We also learned about the `rint()` function that rounds numbers to the closest integer but does not change the type to integer (see `fibonacci.py`):

```
from __future__ import print_function
import numpy as np

F = np.matrix([[1, 1], [1, 0]])
print("F", F)
print("8th Fibonacci", (F ** 7)[0, 0])
n = np.arange(1, 9)

sqrt5 = np.sqrt(5)
phi = (1 + sqrt5)/2
fibonacci = np rint((phi**n - (-1/phi)**n)/sqrt5)
print("Fibonacci", fibonacci)
```

## Have a go hero – timing the calculations

You are probably wondering which approach is faster, so go ahead and time it. Create a universal Fibonacci function with `frompyfunc()` and time that too.

## Lissajous curves

All the standard trigonometric functions such as `sin`, `cos`, `tan`, and so on are represented by universal functions in NumPy (see <https://www.khanacademy.org/math/trigonometry>). **Lissajous curves** are a fun way of using trigonometry. I remember producing Lissajous figures on an oscilloscope in the physics lab. Two parametric equations describe the figures:

```
x = A sin(at + π/2)
y = B sin(bt)
```

## Time for action – drawing Lissajous curves

The Lissajous figures are determined by four parameters: A, B, a, and b. Let's set A and B to 1 for simplicity:

1. Initialize `t` with the `linspace()` function from  $-\pi$  to  $\pi$  with 201 points:

```
a = 9
b = 8
t = np.linspace(-np.pi, np.pi, 201)
```

2. Calculate `x` with the `sin()` function and `np.pi`:

```
x = np.sin(a * t + np.pi/2)
```

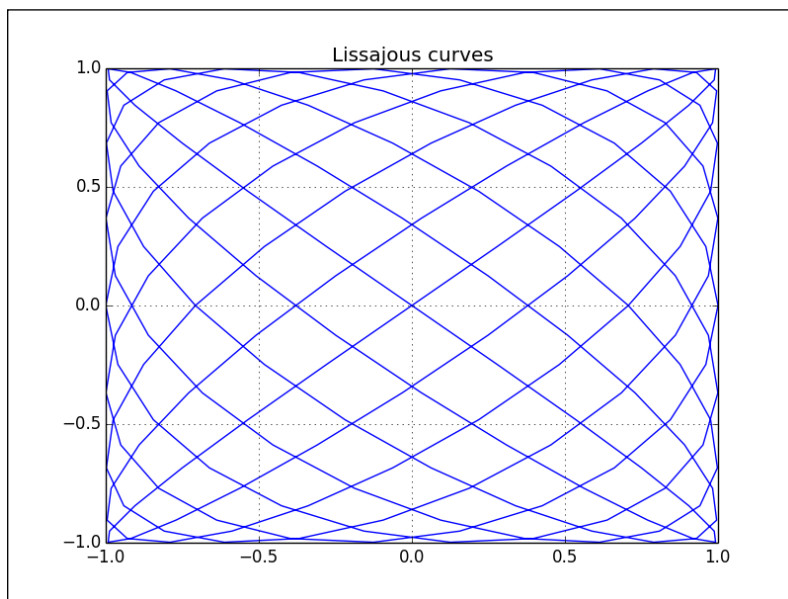
3. Calculate `y` with the `sin()` function:

```
y = np.sin(b * t)
```

4. Plot as shown in the following:

```
plt.plot(x, y)
plt.title('Lissajous curves')
plt.grid()
plt.show()
```

The result for  $a = 9$  and  $b = 8$  is as follows:



## What just happened?

We plotted the Lissajous curve with the aforementioned parametric equations where  $A=B=1$ ,  $a=9$ , and  $b=8$ . We used the `sin()` and `linspace()` functions, as well as the NumPy `pi` constant (see `lissajous.py`):

```
import numpy as np
import matplotlib.pyplot as plt

a = 9
b = 8
t = np.linspace(-np.pi, np.pi, 201)
x = np.sin(a * t + np.pi/2)
y = np.sin(b * t)
plt.plot(x, y)
plt.title('Lissajous curves')
plt.grid()
plt.show()
```

## Square waves

Square waves are also one of those neat things that you can view on an oscilloscope. They can be approximated pretty well with sine waves; after all, a square wave is a signal that can be represented by an infinite **Fourier series**.



A Fourier series is the sum of a series of sine and cosine terms named after the famous mathematician Jean-Baptiste Fourier (see [http://en.wikipedia.org/wiki/Fourier\\_series](http://en.wikipedia.org/wiki/Fourier_series)).

The formula of this particular series representing the square wave is as follows:

$$\sum_{k=1}^{\infty} \frac{4 \sin(2\pi(2k-1)ft)}{(2k-1)\pi}$$

## Time for action – drawing a square wave

We will initialize  $t$  just as in the previous section. We need to sum a number of terms. The higher the number of terms, the more accurate the result;  $k = 99$  should be sufficient. In order to draw a square wave, follow these steps:

1. We will start by initializing  $t$  and  $k$ . Set the initial values for the function to 0:

```
t = np.linspace(-np.pi, np.pi, 201)
k = np.arange(1, 99)
k = 2 * k - 1
f = np.zeros_like(t)
```

2. Compute the function values with the `sin()` and `sum()` functions:

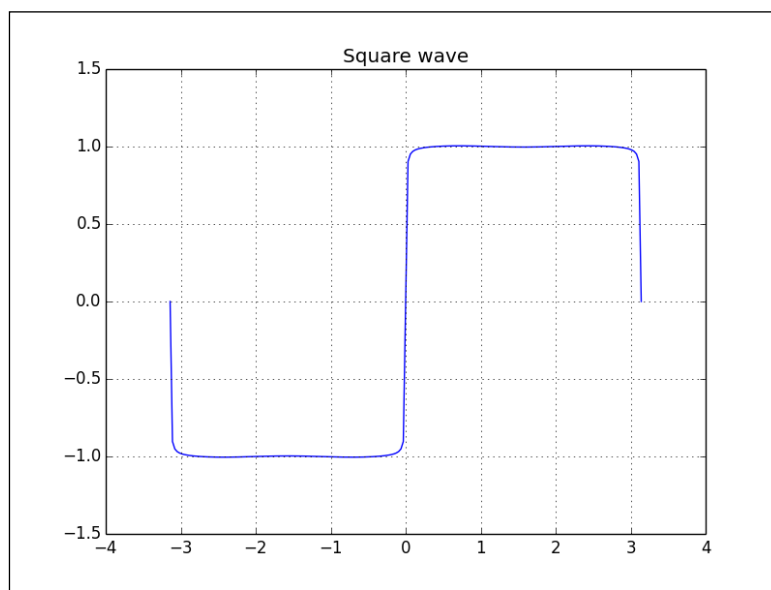
```
for i, ti in enumerate(t):
    f[i] = np.sum(np.sin(k * ti)/k)
```

```
f = (4 / np.pi) * f
```

3. The code to plot is almost identical to the one in the previous section:

```
plt.plot(t, f)
plt.title('Square wave')
plt.grid()
plt.show()
```

The resulting square wave generated with  $k = 99$  is as follows:



## What just happened?

We generated a square wave or, at least, a fair approximation of it, using the `sin()` function. The input values were assembled with the `linspace()` function and the `k` values with the `arange()` function (see `squarewave.py`):

```
import numpy as np
import matplotlib.pyplot as plt

t = np.linspace(-np.pi, np.pi, 201)
k = np.arange(1, 99)
k = 2 * k - 1
f = np.zeros_like(t)

for i, ti in enumerate(t):
    f[i] = np.sum(np.sin(k * ti)/k)

f = (4 / np.pi) * f

plt.plot(t, f)
plt.title('Square wave')
plt.grid()
plt.show()
```

## Have a go hero – getting rid of the loop

You may have noticed that there is one loop in the code. Get rid of it with NumPy functions and make sure the performance is also improved.

## Sawtooth and triangle waves

Sawtooth and triangle waves are also a phenomenon easily viewed on an oscilloscope. Just as with square waves, we can define an infinite Fourier series. The triangle waves can be found by taking the absolute value of a sawtooth wave. The formula for the representation of a series of sawtooth waves is as follows:

$$\sum_{k=1}^{\infty} \frac{-2 \sin(2\pi kft)}{k\pi}$$

## Time for action – drawing sawtooth and triangle waves

We will initialize  $t$  just like in the previous section. Again,  $k = 99$  should be sufficient. In order to draw sawtooth and triangle waves, follow these steps:

1. Set initial values for the function to zero:

```
t = np.linspace(-np.pi, np.pi, 201)
k = np.arange(1, 99)
f = np.zeros_like(t)
```

2. Compute the function values with the `sin()` and `sum()` functions:

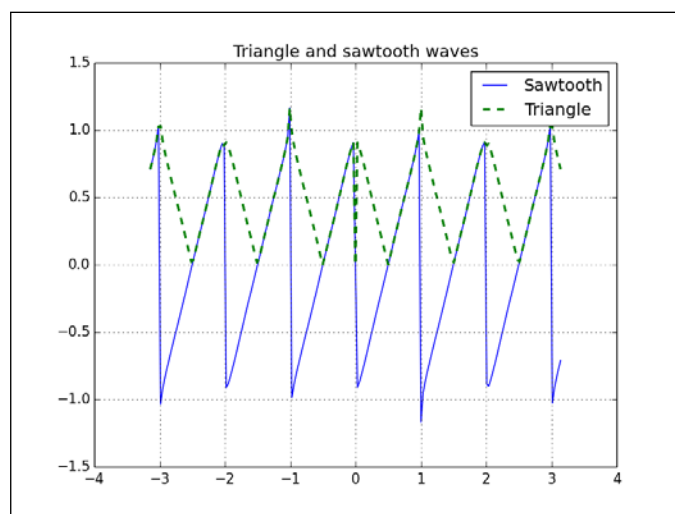
```
for i, ti in enumerate(t):
    f[i] = np.sum(np.sin(2 * np.pi * k * ti)/k)
```

```
f = (-2 / np.pi) * f
```

3. It's easy to plot the sawtooth and triangle waves since the value of the triangle wave should be equal to the absolute value of the sawtooth wave. Plot the waves as shown in the following:

```
plt.plot(t, f, lw=1.0, label='Sawtooth')
plt.plot(t, np.abs(f), '--', lw=2.0, label='Triangle')
plt.title('Triangle and sawtooth waves')
plt.grid()
plt.legend(loc='best')
plt.show()
```

In the following figure, the triangle wave is the one with the dashed line:



## What just happened?

We drew a sawtooth wave using the `sin()` function. We assembled the input values with the `linspace()` function and the `k` values with the `arange()` function. A triangle wave was derived from the sawtooth wave by taking the absolute value (see `sawtooth.py`):

```
import numpy as np
import matplotlib.pyplot as plt

t = np.linspace(-np.pi, np.pi, 201)
k = np.arange(1, 99)
f = np.zeros_like(t)

for i, ti in enumerate(t):
    f[i] = np.sum(np.sin(2 * np.pi * k * ti)/k)

f = (-2 / np.pi) * f
plt.plot(t, f, lw=1.0, label='Sawtooth')
plt.plot(t, np.abs(f), '--', lw=2.0, label='Triangle')
plt.title('Triangle and sawtooth waves')
plt.grid()
plt.legend(loc='best')
plt.show()
```

## Have a go hero – getting rid of the loop

Your challenge, should you choose to accept it, is to get rid of the loop in the program. It should be doable with NumPy functions and the performance should improve.

## Bitwise and comparison functions

Bitwise functions operate on the bits of integers or integer arrays since they are universal functions. The operators `^`, `&`, `|`, `<<`, `>>`, and so on have their NumPy counterparts. The same goes for comparison operators such as `<`, `>`, `==`, and so on. These operators allow you to do clever tricks, which should be good for performance; however, they can make your code quite unreadable, so use them with care.

## Time for action – twiddling bits

We will now cover three tricks—checking whether the signs of integers are different, checking whether a number is a power of 2, and calculating the modulus of a number that is a power of 2. We will show an operators-only notation and one using the corresponding NumPy functions:

1. The first trick depends on the XOR or `^` operator. The XOR operator is also called the inequality operator; so, if the sign bit of the two operands is different, the XOR operation will lead to a negative number (see <https://www.khanacademy.org/computing/computer-science/cryptography/ciphers/a/xor-bitwise-operation>).

The following truth table illustrates the XOR operator:

Input 1	Input 2	XOR
True	True	False
False	True	True
True	False	True
False	False	False

The `^` operator corresponds to the `bitwise_xor()` function, and the `<` operator corresponds to the `less()` function:

```
x = np.arange(-9, 9)
y = -x
print("Sign different?", (x ^ y) < 0)
print("Sign different?", np.less(np.bitwise_xor(x, y), 0))
```

The result is shown as follows:

```
Sign different? [ True  True  True  True  True  True  True  True
 True False  True  True
   True  True  True  True  True  True]
Sign different? [ True  True  True  True  True  True  True  True
 True False  True  True
   True  True  True  True  True  True]
```

As expected, all the signs differ, except for zero.



2. A power of 2 is represented by a 1, followed by a series of trailing zeroes in binary notation. For instance, 10, 100, or 1000. A number one less than a power of 2 will be represented by a row of ones in binary. For instance, 11, 111, or 1111 (or 3, 7, and 15 in the decimal system). Now, if we bitwise AND a power of 2, and the integer that is one less than that, then we should get 0.

The truth table for the AND operator looks like the following:

Input 1	Input 2	AND
True	True	True
False	True	False
True	False	False
False	False	False

The NumPy counterpart of & is `bitwise_and()`, and the counterpart of == is the `equal()` universal function:

```
print("Power of 2?\n", x, "\n", (x & (x - 1)) == 0)
print("Power of 2?\n", x, "\n", np.equal(np.bitwise_and(x,
(x - 1)), 0))
```

The result is shown as follows:

```
Power of 2?
[-9 -8 -7 -6 -5 -4 -3 -2 -1  0  1  2  3  4  5  6  7  8]
[False False False False False False False False False True True
 True
  False True False False False True]
Power of 2?
[-9 -8 -7 -6 -5 -4 -3 -2 -1  0  1  2  3  4  5  6  7  8]
[False False False False False False False False False True True
 True
  False True False False False True]
```

3. The trick of computing the modulus of 4 actually works when taking the modulus of integers that are a power of 2 such as 4, 8, 16, and so on. A bitwise left shift leads to doubling of values (see <https://wiki.python.org/moin/BitwiseOperators>). We saw in the previous step that subtracting one from a power of 2 leads to a number in binary notation that has a row of ones such as 11, 111, or 1111. This basically gives us a mask. Bitwise-ANDing with such a number gives you the remainder with a power of 2. The NumPy equivalent of << is the `left_shift()` universal function:

```
print("Modulus 4\n", x, "\n", x & ((1 << 2) - 1))
```

```
print("Modulus 4\n", x, "\n", np.bitwise_and(x,
      np.left_shift(1, 2) - 1))
```

The result is shown as follows:

```
Modulus 4
[-9 -8 -7 -6 -5 -4 -3 -2 -1  0  1  2  3  4  5  6  7  8]
[3 0 1 2 3 0 1 2 3 0 1 2 3 0 1 2 3 0]
Modulus 4
[-9 -8 -7 -6 -5 -4 -3 -2 -1  0  1  2  3  4  5  6  7  8]
[3 0 1 2 3 0 1 2 3 0 1 2 3 0 1 2 3 0]
```

## What just happened?

We covered three bit twiddling hacks—checking whether the signs of integers are different, checking whether a number is a power of 2, and calculating the modulus of a number that is a power of 2. We saw the NumPy counterparts of the operators `^`, `&`, `<<`, and `<` (see `bittwiddling.py`):

```
from __future__ import print_function
import numpy as np

x = np.arange(-9, 9)
y = -x
print("Sign different?", (x ^ y) < 0)
print("Sign different?", np.less(np.bitwise_xor(x, y), 0))
print("Power of 2?\n", x, "\n", (x & (x - 1)) == 0)
print("Power of 2?\n", x, "\n", np.equal(np.bitwise_and(x, (x - 1)),
      0))
print("Modulus 4\n", x, "\n", x & ((1 << 2) - 1))
print("Modulus 4\n", x, "\n", np.bitwise_and(x, np.left_shift(1, 2)
      - 1))
```

## Fancy indexing

The `at()` method was added in **NumPy 1.8**. This method allows fancy indexing in-place. Fancy indexing is indexing that does not involve integers or slices, which is normal indexing. In-place means that the array we operate on will be modified.

The signature for the `at()` method is `ufunc.at(a, indices[, b])`. The `indices` array specifies the elements to operate on. We need the `b` array only for universal functions with two operands. The following *Time for action* section gives examples of the `at()` method.

## Time for action – fancy indexing in-place for ufuncs with the `at()` method

To demonstrate how the `at()` method works, start a Python or IPython shell and import NumPy. You should know how to do this by now.

1. Create an array with seven random integers from -3 to 3 with a seed of 42:

```
>>> a = np.random.random_integers(-3, 3, 7)
>>> a
array([ 1,  0, -1,  2,  1, -2,  0])
```

When we talk about random numbers in programming, we usually talk about pseudo-random numbers (see <https://www.khanacademy.org/computing/computer-science/cryptography/crypt/v/random-vs-pseudorandom-number-generators>). The numbers appear random, but in fact are calculated using a seed.

2. Apply the `at()` method of the `sign()` universal function to the fourth and sixth array elements:

```
>>> np.sign.at(a, [3, 5])
>>> a
array([ 1,  0, -1,  1,  1, -1,  0])
```

### ***What just happened?***

We used the `at()` method to select array elements and performed an in-place operation—determining the sign. We also learned how to create random integers.

## Summary

In this chapter, you learned, about matrices and universal functions. We covered how to create matrices and looked at how universal functions work. You had a brief introduction to arithmetic, trigonometric, bitwise, and comparison universal functions.

In the next chapter, you will cover the NumPy modules.

# 6

## Moving Further with NumPy Modules

*NumPy has a number of modules inherited from its predecessor, Numeric. Some of these packages have a SciPy counterpart, which may have fuller functionality. We will discuss SciPy in a later chapter.*

In this chapter, we will cover the following topics:

- ◆ The `linalg` package
- ◆ The `fft` package
- ◆ Random numbers
- ◆ Continuous and discrete distributions

### Linear algebra

Linear algebra is an important branch of mathematics. The `numpy.linalg` package contains linear algebra functions. With this module, you can invert matrices, calculate eigenvalues, solve linear equations, and determine determinants, among other things (see <http://docs.scipy.org/doc/numpy/reference/routines.linalg.html>).

## Time for action – inverting matrices

The inverse of a matrix  $A$  in linear algebra is the matrix  $A^{-1}$ , which, when multiplied with the original matrix, is equal to the identity matrix  $I$ . This can be written as follows:

$$A A^{-1} = I$$

The `inv()` function in the `numpy.linalg` package can invert an example matrix with the following steps:

1. Create the example matrix with the `mat()` function we used in the previous chapters:

```
A = np.mat("0 1 2;1 0 3;4 -3 8")
print("A\n", A)
```

The  $A$  matrix appears as follows:

```
A
[[ 0  1  2]
 [ 1  0  3]
 [ 4 -3  8]]
```

2. Invert the matrix with the `inv()` function:

```
inverse = np.linalg.inv(A)
print("inverse of A\n", inverse)
```

The inverse matrix appears as follows:

```
inverse of A
[[-4.5  7.  -1.5]
 [-2.   4.  -1. ]
 [ 1.5 -2.   0.5]]
```



If the matrix is singular, or not square, a `LinAlgError` is raised. If you want, you can check the result manually with a pen and paper. This is left as an exercise for the reader.

3. Check the result by multiplying the original matrix with the result of the `inv()` function:

```
print("Check\n", A * inverse)
```

The result is the identity matrix, as expected:

Check

```
[[ 1.  0.  0.]  
 [ 0.  1.  0.]  
 [ 0.  0.  1.]]
```

### ***What just happened?***

We calculated the inverse of a matrix with the `inv()` function of the `numpy.linalg` package. We checked, with matrix multiplication, whether this is indeed the inverse matrix (see `inversion.py`):

```
from __future__ import print_function  
import numpy as np  
  
A = np.mat("0 1 2;1 0 3;4 -3 8")  
print("A\n", A)  
  
inverse = np.linalg.inv(A)  
print("inverse of A\n", inverse)  
  
print("Check\n", A * inverse)
```

### **Pop quiz – creating a matrix**

Q1. Which function can create matrices?

1. `array`
2. `create_matrix`
3. `mat`
4. `vector`

### **Have a go hero – inverting your own matrix**

Create your own matrix and invert it. The inverse is only defined for square matrices. The matrix must be square and invertible; otherwise, a `LinAlgError` exception is raised.

## Solving linear systems

A matrix transforms a vector into another vector in a linear way. This transformation mathematically corresponds to a system of linear equations. The `numpy.linalg` function `solve()` solves systems of linear equations of the form  $Ax = b$ , where  $A$  is a matrix,  $b$  can be a one-dimensional or two-dimensional array, and  $x$  is an unknown variable. We will see the `dot()` function in action. This function returns the dot product of two floating-point arrays.

The `dot()` function calculates the dot product (see [https://www.khanacademy.org/math/linear-algebra/vectors\\_and\\_spaces/dot\\_cross\\_products/v/vector-dot-product-and-vector-length](https://www.khanacademy.org/math/linear-algebra/vectors_and_spaces/dot_cross_products/v/vector-dot-product-and-vector-length)). For a matrix  $A$  and vector  $b$ , the dot product is equal to the following sum:

$$\sum_i A_i B_i$$

### Time for action – solving a linear system

Solve an example of a linear system with the following steps:

1. Create  $A$  and  $b$ :

```
A = np.mat("1 -2 1;0 2 -8;-4 5 9")
print("A\n", A)
b = np.array([0, 8, -9])
print("b\n", b)
```

$A$  and  $b$  appear as follows:

```
A
[[ 1 -2  1]
 [ 0  2 -8]
 [-4  5  9]]
b
[ 0  8 -9]
```

2. Solve this linear system with the `solve()` function:

```
x = np.linalg.solve(A, b)
print("Solution", x)
```

The solution of the linear system is as follows:

```
Solution [ 29.  16.   3.]
```

3. Check whether the solution is correct with the `dot()` function:

```
print("Check\n", np.dot(A , x))
```

The result is as expected:

**Check**

```
[[ 0.  8. -9.]]
```

### ***What just happened?***

We solved a linear system using the `solve()` function from the NumPy `linalg` module and checked the solution with the `dot()` function. Please refer to the `solution.py` file in this book's code bundle:

```
from __future__ import print_function
import numpy as np

A = np.mat("1 -2 1;0 2 -8;-4 5 9")
print("A\n", A)

b = np.array([0, 8, -9])
print("b\n", b)

x = np.linalg.solve(A, b)
print("Solution", x)

print("Check\n", np.dot(A , x))
```

## **Finding eigenvalues and eigenvectors**

**Eigenvalues** are scalar solutions to the equation  $Ax = \lambda x$ , where  $A$  is a two-dimensional matrix and  $x$  is a one-dimensional vector. **Eigenvectors** are vectors corresponding to eigenvalues (see [https://www.khanacademy.org/math/linear-algebra/alternate\\_bases/eigen\\_everything/v/linear-algebra-introduction-to-eigenvalues-and-eigenvectors](https://www.khanacademy.org/math/linear-algebra/alternate_bases/eigen_everything/v/linear-algebra-introduction-to-eigenvalues-and-eigenvectors)). The `eigvals()` function in the `numpy.linalg` package calculates eigenvalues. The `eig()` function returns a tuple containing eigenvalues and eigenvectors.



## Time for action – determining eigenvalues and eigenvectors

Let's calculate the eigenvalues of a matrix:

1. Create a matrix as shown in the following:

```
A = np.mat("3 -2;1 0")
print("A\n", A)
```

The matrix we created looks like the following:

```
A
[[ 3 -2]
 [ 1  0]]
```

2. Call the `eigvals()` function:

```
print("Eigenvalues", np.linalg.eigvals(A))
```

The eigenvalues of the matrix are as follows:

```
Eigenvalues [ 2.  1.]
```

3. Determine eigenvalues and eigenvectors with the `eig()` function. This function returns a tuple, where the first element contains eigenvalues and the second element contains corresponding eigenvectors, arranged column-wise:

```
eigenvalues, eigenvectors = np.linalg.eig(A)
print("First tuple of eig", eigenvalues)
print("Second tuple of eig\n", eigenvectors)
```

The eigenvalues and eigenvectors appear as follows:

```
First tuple of eig [ 2.  1.]
Second tuple of eig
[[ 0.89442719  0.70710678]
 [ 0.4472136  0.70710678]]
```

4. Check the result with the `dot()` function by calculating the right and left side of the eigenvalues equation  $Ax = ax$ :

```
for i, eigenvalue in enumerate(eigenvalues):
    print("Left", np.dot(A, eigenvectors[:,i]))
    print("Right", eigenvalue * eigenvectors[:,i])
    print()
```

The output is as follows:

```
Left [[ 1.78885438]
      [ 0.89442719]]
Right [[ 1.78885438]
       [ 0.89442719]]
```

### ***What just happened?***

We found the eigenvalues and eigenvectors of a matrix with the `eigvals()` and `eig()` functions of the `numpy.linalg` module. We checked the result using the `dot()` function (see `eigenvalues.py`):

```
from __future__ import print_function
import numpy as np

A = np.mat("3 -2;1 0")
print("A\n", A)

print("Eigenvalues", np.linalg.eigvals(A) )

eigenvalues, eigenvectors = np.linalg.eig(A)
print("First tuple of eig", eigenvalues)
print("Second tuple of eig\n", eigenvectors)

for i, eigenvalue in enumerate(eigenvalues):
    print("Left", np.dot(A, eigenvectors[:,i]))
    print("Right", eigenvalue * eigenvectors[:,i])
    print()
```

## **Singular value decomposition**

**Singular value decomposition (SVD)** is a type of factorization that decomposes a matrix into a product of three matrices. The SVD is a generalization of the previously discussed eigenvalue decomposition. SVD is very useful for algorithms such as the pseudo inverse, which we will discuss in the next section. The `svd()` function in the `numpy.linalg` package can perform this decomposition. This function returns three matrices  $U$ ,  $\Sigma$ , and  $V$  such that  $U$  and  $V$  are unitary and  $\Sigma$  contains the singular values of the input matrix:

$$M = U \Sigma V^*$$

The asterisk denotes the **Hermitian conjugate** or the **conjugate transpose**. The **complex conjugate** changes the sign of the imaginary part of a complex number and is therefore not relevant for real numbers.



A complex square matrix  $A$  is unitary if  $A^*A = AA^* = I$  (the identity matrix). We can interpret SVD as a sequence of three operations—rotation, scaling, and another rotation.

We already transposed matrices in this book. The transpose flips matrices, turning rows into columns, and columns into rows.

## Time for action – decomposing a matrix

It's time to decompose a matrix with the SVD using the following steps:

1. First, create a matrix as shown in the following:

```
A = np.mat("4 11 14;8 7 -2")
print("A\n", A)
```

The matrix we created looks like the following:

```
A
[[ 4 11 14]
 [ 8  7 -2]]
```

2. Decompose the matrix with the `svd()` function:

```
U, Sigma, V = np.linalg.svd(A, full_matrices=False)
print("U")
print(U)
print("Sigma")
print(Sigma)
print("V")
print(V)
```

Because of the `full_matrices=False` specification, NumPy performs a reduced SVD decomposition, which is faster to compute. The result is a tuple containing the two unitary matrices  $U$  and  $V$  on the left and right, respectively, and the singular values of the middle matrix:

```
U
[[-0.9486833  -0.31622777]
 [-0.31622777  0.9486833  ]]
```

```

Sigma
[ 18.97366596  9.48683298]
V
[[-0.33333333 -0.66666667 -0.66666667]
 [ 0.66666667  0.33333333 -0.66666667]]

```

3. We do not actually have the middle matrix—we only have the diagonal values. The other values are all 0. Form the middle matrix with the `diag()` function. Multiply the three matrices as follows:

```
print("Product\n", U * np.diag(Sigma) * V)
```

The product of the three matrices is equal to the matrix we created in the first step:

```

Product
[[ 4.  11.  14.]
 [ 8.   7.  -2.]]

```

### ***What just happened?***

We decomposed a matrix and checked the result by matrix multiplication. We used the `svd()` function from the NumPy `linalg` module (see `decomposition.py`):

```

from __future__ import print_function
import numpy as np

A = np.mat("4 11 14;8 7 -2")
print("A\n", A)

U, Sigma, V = np.linalg.svd(A, full_matrices=False)

print("U")
print(U)

print("Sigma")
print(Sigma)

print("V")
print(V)

print("Product\n", U * np.diag(Sigma) * V)

```

## Pseudo inverse

The **Moore-Penrose pseudo inverse** of a matrix can be computed with the `pinv()` function of the `numpy.linalg` module (see [http://en.wikipedia.org/wiki/Moore%E2%80%93Penrose\\_pseudoinverse](http://en.wikipedia.org/wiki/Moore%E2%80%93Penrose_pseudoinverse)). The pseudo inverse is calculated using the SVD (see previous example). The `inv()` function only accepts square matrices; the `pinv()` function does not have this restriction and is therefore considered a generalization of the inverse.

### Time for action – computing the pseudo inverse of a matrix

Let's compute the pseudo inverse of a matrix:

1. First, create a matrix:

```
A = np.mat("4 11 14;8 7 -2")
print("A\n", A)
```

The matrix we created looks like the following:

```
A
[[ 4 11 14]
 [ 8  7 -2]]
```

2. Calculate the pseudo inverse matrix with the `pinv()` function:

```
pseudoinv = np.linalg.pinv(A)
print("Pseudo inverse\n", pseudoinv)
```

The pseudo inverse result is as follows:

```
Pseudo inverse
[[-0.00555556  0.07222222]
 [ 0.02222222  0.04444444]
 [ 0.05555556 -0.05555556]]
```

3. Multiply the original and pseudo inverse matrices:

```
print("Check", A * pseudoinv)
```

What we get is not an identity matrix, but it comes close to it:

```
Check [[ 1.00000000e+00  0.00000000e+00]
 [ 8.32667268e-17  1.00000000e+00]]
```

## What just happened?

We computed the pseudo inverse of a matrix with the `pinv()` function of the `numpy.linalg` module. The check by matrix multiplication resulted in a matrix that is approximately an identity matrix (see `pseudoinversion.py`):

```
from __future__ import print_function
import numpy as np

A = np.mat("4 11 14;8 7 -2")
print("A\n", A)

pseudoinv = np.linalg.pinv(A)
print("Pseudo inverse\n", pseudoinv)

print("Check", A * pseudoinv)
```

## Determinants

The **determinant** is a value associated with a square matrix. It is used throughout mathematics; for more details, please refer to <http://en.wikipedia.org/wiki/Determinant>. For a  $n \times n$  real value matrix, the determinant corresponds to the scaling a  $n$ -dimensional volume undergoes when transformed by the matrix. The positive sign of the determinant means the volume preserves its orientation (clockwise or anticlockwise), while a negative sign means reversed orientation. The `numpy.linalg` module has a `det()` function that returns the determinant of a matrix.

### Time for action – calculating the determinant of a matrix

To calculate the determinant of a matrix, follow these steps:

1. Create the matrix:

```
A = np.mat("3 4;5 6")
print("A\n", A)
```

The matrix we created appears as follows:

```
A
[[ 3.  4.]
 [ 5.  6.]
```

2. Compute the determinant with the `det()` function:

```
print("Determinant", np.linalg.det(A))
```

The determinant appears as follows:

```
Determinant -2.0
```

## What just happened?

We calculated the determinant of a matrix with the `det()` function from the `numpy.linalg` module (see `determinant.py`):

```
from __future__ import print_function
import numpy as np

A = np.mat("3 4;5 6")
print("A\n", A)

print("Determinant", np.linalg.det(A))
```

## Fast Fourier transform

The **Fast Fourier transform (FFT)** is an efficient algorithm to calculate the **discrete Fourier transform (DFT)**.



The Fourier transform is related to the **Fourier series**, which was mentioned in the previous chapter—*Chapter 5, Working with Matrices and ufuncs*. The Fourier series represents a signal as a sum of sine and cosine terms.

FFT improves on more naïve algorithms and is of order  $O(N \log N)$ . DFT has applications in signal processing, image processing, solving partial differential equations, and more. NumPy has a module called `fft` that offers FFT functionality. Many functions in this module are paired; for those functions, another function does the inverse operation. For instance, the `fft()` and `ifft()` function form such a pair.

## Time for action – calculating the Fourier transform

First, we will create a signal to transform. Calculate the Fourier transform with the following steps:

1. Create a cosine wave with 30 points as follows:

```
x = np.linspace(0, 2 * np.pi, 30)
wave = np.cos(x)
```

2. Transform the cosine wave with the `fft()` function:

```
transformed = np.fft.fft(wave)
```

3. Apply the inverse transform with the `ifft()` function. It should approximately return the original signal. Check with the following line:

```
print(np.all(np.abs(np.fft.ifft(transformed) - wave)  
       < 10 ** -9))
```

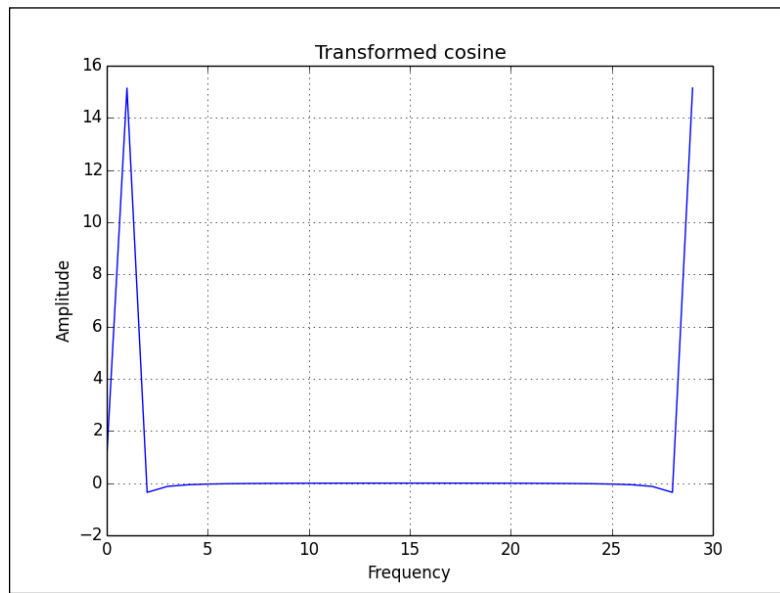
The result appears as follows:

**True**

4. Plot the transformed signal with `matplotlib`:

```
plt.plot(transformed)  
plt.title('Transformed cosine')  
plt.xlabel('Frequency')  
plt.ylabel('Amplitude')  
plt.grid()  
plt.show()
```

The following resulting diagram shows the FFT result:





## What just happened?

We applied the `fft()` function to a cosine wave. After applying the `ifft()` function, we got our signal back (see `fourier.py`):

```
from __future__ import print_function
import numpy as np
import matplotlib.pyplot as plt

x = np.linspace(0, 2 * np.pi, 30)
wave = np.cos(x)
transformed = np.fft.fft(wave)
print(np.all(np.abs(np.fft.ifft(transformed) - wave) < 10 ** -9))

plt.plot(transformed)
plt.title('Transformed cosine')
plt.xlabel('Frequency')
plt.ylabel('Amplitude')
plt.grid()
plt.show()
```

## Shifting

The `fftshift()` function of the `numpy.linalg` module shifts zero-frequency components to the center of a spectrum. The zero-frequency component corresponds to the mean of the signal. The `ifftshift()` function reverses this operation.

### Time for action – shifting frequencies

We will create a signal, transform it, and then shift the signal. Shift the frequencies with the following steps:

1. Create a cosine wave with 30 points:

```
x = np.linspace(0, 2 * np.pi, 30)
wave = np.cos(x)
```
2. Transform the cosine wave with the `fft()` function:

```
transformed = np.fft.fft(wave)
```

3. Shift the signal with the `fftshift()` function:

```
shifted = np.fft.fftshift(transformed)
```

4. Reverse the shift with the `ifftshift()` function. This should undo the shift. Check with the following code snippet:

```
print(np.all((np.fft.ifftshift(shifted) - transformed)
           < 10 ** -9))
```

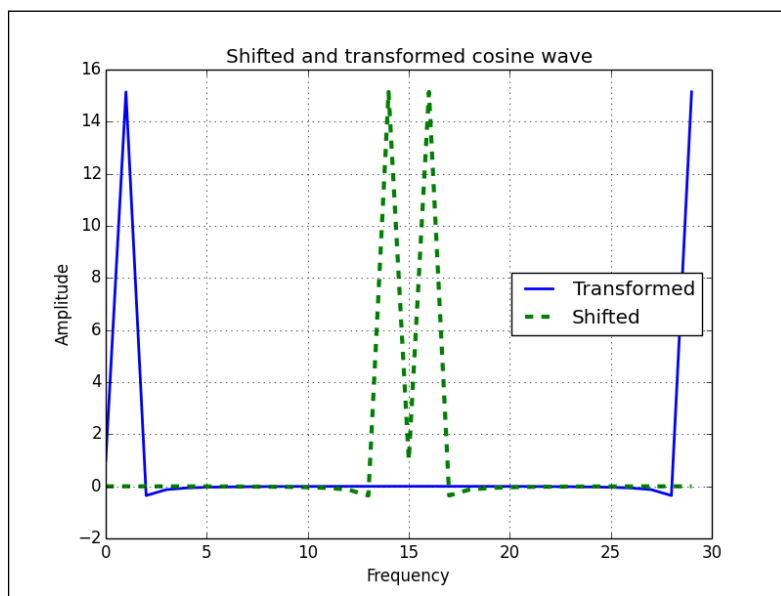
The result appears as follows:

**True**

5. Plot the signal and transform it with matplotlib:

```
plt.plot(transformed, lw=2, label="Transformed")
plt.plot(shifted, '--', lw=3, label="Shifted")
plt.title('Shifted and transformed cosine wave')
plt.xlabel('Frequency')
plt.ylabel('Amplitude')
plt.grid()
plt.legend(loc='best')
plt.show()
```

The following diagram shows the effect of the shift and the FFT:



## What just happened?

We applied the `fftshift()` function to a cosine wave. After applying the `ifftshift()` function, we got our signal back (see `fouriershift.py`):

```
import numpy as np
import matplotlib.pyplot as plt

x = np.linspace(0, 2 * np.pi, 30)
wave = np.cos(x)
transformed = np.fft.fft(wave)
shifted = np.fft.fftshift(transformed)
print(np.all(np.abs(np.fft.ifftshift(shifted) - transformed) < 10 **
-9))

plt.plot(transformed, lw=2, label="Transformed")
plt.plot(shifted, '--', lw=3, label="Shifted")
plt.title('Shifted and transformed cosine wave')
plt.xlabel('Frequency')
plt.ylabel('Amplitude')
plt.grid()
plt.legend(loc='best')
plt.show()
```

## Random numbers

Random numbers are used in Monte Carlo methods, stochastic calculus, and more. Real random numbers are hard to generate, so, in practice, we use **pseudo random numbers**, which are random enough for most intents and purposes, except for some very special cases. These numbers appear random, but if you analyze them more closely, you will realize that they follow a certain pattern. The random numbers-related functions are in the NumPy random module. The core random number generator is based on the **Mersenne Twister algorithm**—a standard and well-known algorithm (see [https://en.wikipedia.org/wiki/Mersenne\\_Twister](https://en.wikipedia.org/wiki/Mersenne_Twister)). We can generate random numbers from discrete or continuous distributions. The distribution functions have an optional size parameter, which tells NumPy how many numbers to generate. You can specify either an integer or a tuple as size. This will result in an array filled with random numbers of appropriate shape. Discrete distributions include the geometric, hypergeometric, and binomial distributions.

## Time for action – gambling with the binomial

The **binomial distribution** models the number of successes in an integer number of independent trials of an experiment, where the probability of success in each experiment is a fixed number (see [https://www.khanacademy.org/math/probability/random-variables-topic/binomial\\_distribution](https://www.khanacademy.org/math/probability/random-variables-topic/binomial_distribution)).

Imagine a 17th century gambling house where you can bet on flipping pieces of eight. Nine coins are flipped. If less than five are heads, then you lose one piece of eight, otherwise you win one. Let's simulate this, starting with 1,000 coins in our possession. Use the `binomial()` function from the `random` module for that purpose.

To understand the `binomial()` function, look at the following section:

1. Initialize an array, which represents the cash balance, to zeros. Call the `binomial()` function with a size of 10000. This represents 10,000 coin flips in our casino:

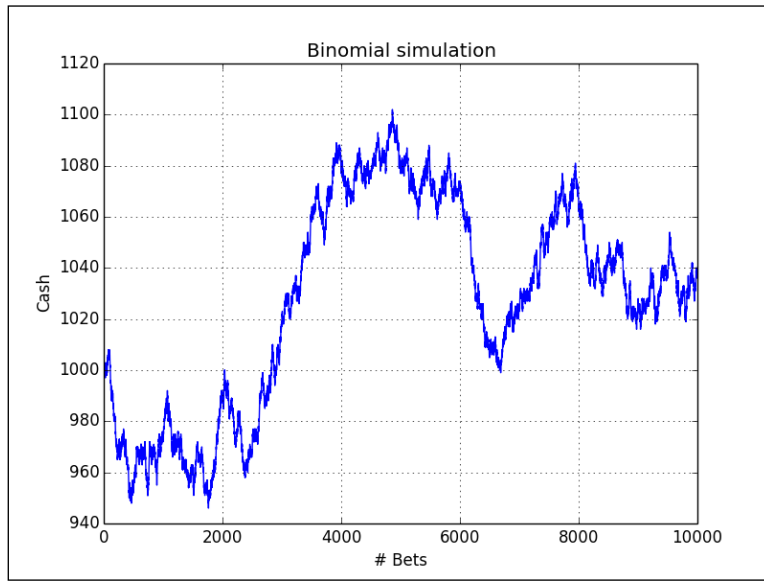
```
cash = np.zeros(10000)
cash[0] = 1000
outcome = np.random.binomial(9, 0.5, size=len(cash))
```

2. Go through the outcomes of the coin flips and update the cash array. Print the minimum and maximum of the outcome, just to make sure we don't have any strange outliers:

```
for i in range(1, len(cash)):
    if outcome[i] < 5:
        cash[i] = cash[i - 1] - 1
    elif outcome[i] < 10:
        cash[i] = cash[i - 1] + 1
    else:
        raise AssertionError("Unexpected outcome " + outcome)

print(outcome.min(), outcome.max())
```

As expected, the values are between 0 and 9. In the following diagram, you can see the cash balance performing a random walk:



### ***What just happened?***

We did a random walk experiment using the `binomial()` function from the NumPy random module (see `headortail.py`):

```
from __future__ import print_function
import numpy as np
import matplotlib.pyplot as plt

cash = np.zeros(10000)
cash[0] = 1000
np.random.seed(73)
outcome = np.random.binomial(9, 0.5, size=len(cash))

for i in range(1, len(cash)):
    if outcome[i] < 5:
        cash[i] = cash[i - 1] - 1
    elif outcome[i] < 10:
        cash[i] = cash[i - 1] + 1
    else:
```

```

        raise AssertionError("Unexpected outcome " + outcome)

print(outcome.min(), outcome.max())

plt.plot(np.arange(len(cash)), cash)
plt.title('Binomial simulation')
plt.xlabel('# Bets')
plt.ylabel('Cash')
plt.grid()
plt.show()

```

## Hypergeometric distribution

The **hypergeometric distribution** models a jar with two types of objects in it. The model tells us how many objects of one type we can get if we take a specified number of items out of the jar without replacing them (see [https://en.wikipedia.org/wiki/Hypergeometric\\_distribution](https://en.wikipedia.org/wiki/Hypergeometric_distribution)). The NumPy random module has a `hypergeometric()` function that simulates this situation.

### Time for action – simulating a game show

Imagine a game show where every time the contestants answer a question correctly, they get to pull three balls from a jar and then put them back. Now, there is a catch, one ball in the jar is bad. Every time it is pulled out, the contestants lose six points. If, however, they manage to get out 3 of the 25 normal balls, they get one point. So, what is going to happen if we have 100 questions in total? Look at the following section for the solution:

1. Initialize the outcome of the game with the `hypergeometric()` function. The first parameter of this function is the number of ways to make a good selection, the second parameter is the number of ways to make a bad selection, and the third parameter is the number of items sampled:

```

points = np.zeros(100)
outcomes = np.random.hypergeometric(25, 1, 3, size=len(points))

```

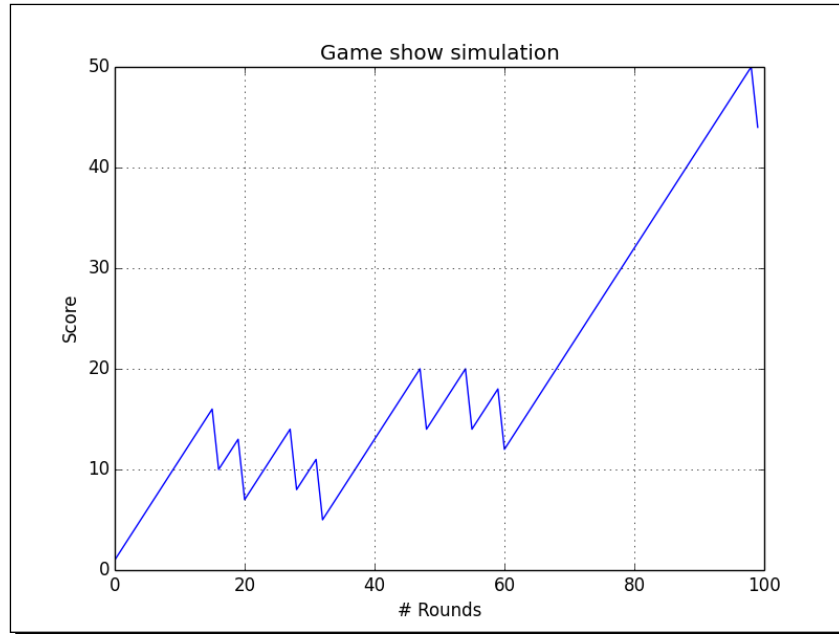
2. Set the scores based on the outcomes from the previous step:

```

for i in range(len(points)):
    if outcomes[i] == 3:
        points[i] = points[i - 1] + 1
    elif outcomes[i] == 2:
        points[i] = points[i - 1] - 6
    else:
        print(outcomes[i])

```

The following diagram shows how the scoring evolved:



### ***What just happened?***

We simulated a game show using the `hypergeometric()` function from the NumPy `random` module. The game scoring depends on how many good and how many bad balls the contestants pulled out of a jar in each session (see `urn.py`):

```
from __future__ import print_function
import numpy as np
import matplotlib.pyplot as plt

points = np.zeros(100)
np.random.seed(16)
outcomes = np.random.hypergeometric(25, 1, 3, size=len(points))

for i in range(len(points)):
    if outcomes[i] == 3:
        points[i] = points[i - 1] + 1
    elif outcomes[i] == 2:
        points[i] = points[i - 1] - 6
    else:
```

```

print(outcomes[i])

plt.plot(np.arange(len(points)), points)
plt.title('Game show simulation')
plt.xlabel('# Rounds')
plt.ylabel('Score')
plt.grid()
plt.show()

```

## Continuous distributions

We usually model continuous distributions with **probability density functions (PDF)**. The probability that a value is in a certain interval is determined by integration of the PDF (see [https://www.khanacademy.org/math/probability/random-variables-topic/random\\_variables\\_prob\\_dist/v/probability-density-functions](https://www.khanacademy.org/math/probability/random-variables-topic/random_variables_prob_dist/v/probability-density-functions)). The NumPy random module has functions that represent continuous distributions—`beta()`, `chisquare()`, `exponential()`, `f()`, `gamma()`, `gumbel()`, `laplace()`, `lognormal()`, `logistic()`, `multivariate_normal()`, `noncentral_chisquare()`, `noncentral_f()`, `normal()`, and others.

### Time for action – drawing a normal distribution

We can generate random numbers from a normal distribution and visualize their distribution with a histogram (see [https://www.khanacademy.org/math/probability/statistics-inferential/normal\\_distribution/v/introduction-to-the-normal-distribution](https://www.khanacademy.org/math/probability/statistics-inferential/normal_distribution/v/introduction-to-the-normal-distribution)). Draw a normal distribution with the following steps:

1. Generate random numbers for a given sample size using the `normal()` function from the `random` NumPy module:

```

N=10000
normal_values = np.random.normal(size=N)

```

2. Draw the histogram and theoretical PDF with a center value of 0 and standard deviation of 1. Use `matplotlib` for this purpose:

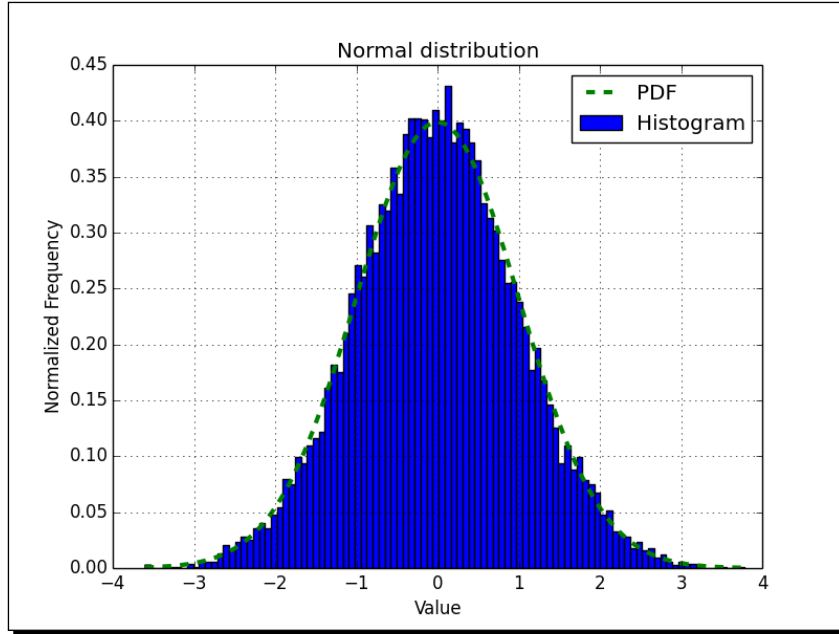
```

_, bins, _ = plt.hist(normal_values,
    np.sqrt(N), normed=True, lw=1)
sigma = 1
mu = 0
plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi))
    * np.exp(- (bins - mu)**2 / (2 * sigma**2) ),lw=2)
plt.show()

```



In the following diagram, we see the familiar bell curve:



### ***What just happened?***

We visualized the normal distribution using the `normal()` function from the random NumPy module. We did this by drawing the bell curve and a histogram of randomly generated values (see `normaldist.py`):

```
import numpy as np
import matplotlib.pyplot as plt

N=10000

np.random.seed(27)
normal_values = np.random.normal(size=N)
_, bins, _ = plt.hist(normal_values, np.sqrt(N), normed=True, lw=1,
label="Histogram")
sigma = 1
mu = 0
```

```
plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) * np.exp( - (bins -
mu)**2 / (2 * sigma**2) ), '--', lw=3, label="PDF")
plt.title('Normal distribution')
plt.xlabel('Value')
plt.ylabel('Normalized Frequency')
plt.grid()
plt.legend(loc='best')
plt.show()
```

## Lognormal distribution

A **lognormal** distribution is a distribution of a random variable whose natural logarithm is normally distributed. The `lognormal()` function of the `random NumPy` module models this distribution.

### Time for action – drawing the lognormal distribution

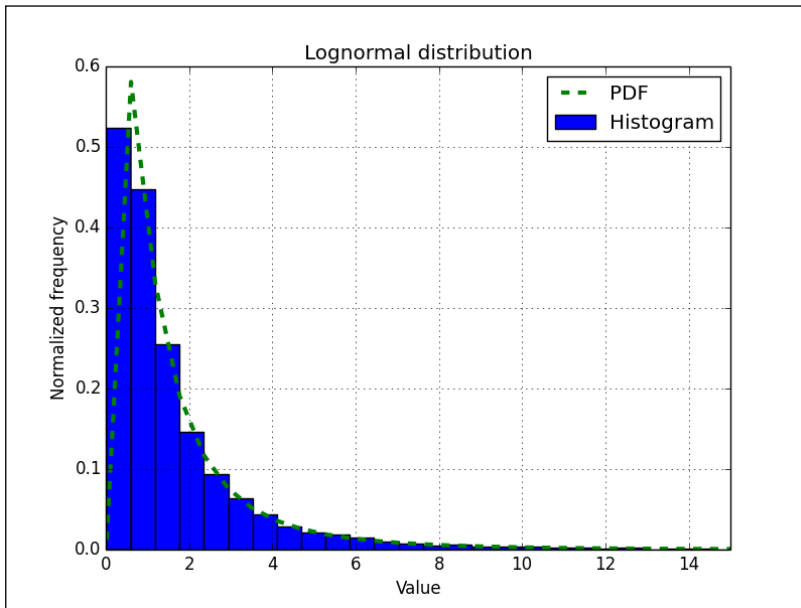
Let's visualize the lognormal distribution and its PDF with a histogram:

1. Generate random numbers using the `normal()` function from the `random NumPy` module:
2. Draw the histogram and theoretical PDF with a center value of 0 and standard deviation of 1:

```
N=10000
lognormal_values = np.random.lognormal(size=N)

_, bins, _ = plt.hist(lognormal_values,
np.sqrt(N), normed=True, lw=1)
sigma = 1
mu = 0
x = np.linspace(min(bins), max(bins), len(bins))
pdf = np.exp(-(numpy.log(x) - mu)**2 / (2 * sigma**2)) / (x *
sigma * np.sqrt(2 * np.pi))
plt.plot(x, pdf, lw=3)
plt.show()
```

The fit of the histogram and theoretical PDF is excellent, as you can see in the following diagram:



### ***What just happened?***

We visualized the lognormal distribution using the `lognormal()` function from the `random` NumPy module. We did this by drawing the curve of the theoretical PDF and a histogram of randomly generated values (see `lognormaldist.py`):

```
import numpy as np
import matplotlib.pyplot as plt

N=10000
np.random.seed(34)
lognormal_values = np.random.lognormal(size=N)
_, bins, _ = plt.hist(lognormal_values,
    np.sqrt(N), normed=True, lw=1, label="Histogram")
sigma = 1
mu = 0
x = np.linspace(min(bins), max(bins), len(bins))
pdf = np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2)) / (x * sigma *
    np.sqrt(2 * np.pi))
plt.xlim([0, 15])
plt.plot(x, pdf, '--', lw=3, label="PDF")
```

```
plt.title('Lognormal distribution')
plt.xlabel('Value')
plt.ylabel('Normalized frequency')
plt.grid()
plt.legend(loc='best')
plt.show()
```

## Bootstrapping in statistics

**Bootstrapping** is a method used to estimate variance, accuracy, and other metrics of sample estimates, such as the arithmetic mean. The simplest bootstrapping procedure consists of the following steps:

1. Generate a large number of samples from the original data sample having the same size  $N$ . You can think of the original data as a jar containing numbers. We create the new samples by  $N$  times randomly picking a number from the jar. Each time we return the number into the jar, so a number can occur multiple times in a generated sample.
2. With the new samples, we calculate the statistical estimate under investigation for each sample (for example, the arithmetic mean). This gives us a sample of possible values for the estimator.

### Time for action – sampling with `numpy.random.choice()`

We will use the `numpy.random.choice()` function to perform bootstrapping.

1. Start the IPython or Python shell and import NumPy:

```
$ ipython
In [1]: import numpy as np
```

2. Generate a data sample following the normal distribution:

```
In [2]: N = 500

In [3]: np.random.seed(52)
```

```
In [4]: data = np.random.normal(size=N)
```

3. Calculate the mean of the data:

```
In [5]: data.mean()
Out[5]: 0.07253250605445645
```

Generate 100 samples from the original data and calculate their means (of course, more samples may lead to a more accurate result):

```
In [6]: bootstrapped = np.random.choice(data, size=(N, 100))
```

```
In [7]: means = bootstrapped.mean(axis=0)
```

```
In [8]: means.shape
```

```
Out[8]: (100,)
```

4. Calculate the mean, variance, and standard deviation of the arithmetic means we obtained:

```
In [9]: means.mean()
```

```
Out[9]: 0.067866373318115278
```

```
In [10]: means.var()
```

```
Out[10]: 0.001762807104774598
```

```
In [11]: means.std()
```

```
Out[11]: 0.041985796464692651
```

If we are assuming a normal distribution for the means, it may be relevant to know the **z-score**, which is defined as follows:

$$z = \frac{x - \mu}{\sigma}$$

```
In [12]: (data.mean() - means.mean())/means.std()
```

```
Out[12]: 0.11113598238549766
```

From the z-score value, we get an idea of how probable the actual mean is.

### ***What just happened?***

We bootstrapped a data sample by generating samples and calculating the means of each sample. Then we computed the mean, standard deviation, variance, and z-score of the means. We used the `numpy.random.choice()` function for bootstrapping.

### **Summary**

You learned a lot in this chapter about NumPy modules. We covered linear algebra, the Fast Fourier transform, continuous and discrete distributions, and random numbers.

In the next chapter, we will cover specialized routines. These are functions that you probably will not use often, but are very useful when you do need them.



# 7

## Peeking into Special Routines

*As NumPy users, we sometimes find ourselves having special needs, for instance, financial calculations or signal processing. Fortunately, NumPy provides for most of our needs. This chapter describes some of the more specialized NumPy functions.*

In this chapter, we will cover the following topics:

- ◆ Sorting and searching
- ◆ Special functions
- ◆ Financial utilities
- ◆ Window functions

### Sorting

NumPy has several data sorting routines:

- ◆ The `sort()` function returns a sorted array
- ◆ The `lexsort()` function performs sorting with a list of keys
- ◆ The `argsort()` function returns the indices that will sort an array
- ◆ The `ndarray` class has a `sort()` method that performs in-place sorting
- ◆ The `msort()` function sorts an array along the first axis
- ◆ The `sort_complex()` function sorts complex numbers by their real part and then their imaginary part



From this list, the `argsort()` and `sort()` functions are available as methods on NumPy arrays as well.

## Time for action – sorting lexically

The NumPy `lexsort()` function returns an array of indices of the input array elements corresponding to lexically sorting an array. We need to give the function an array or tuple of sort keys:

1. Let's go back to *Chapter 3, Getting Familiar with Commonly Used Functions*. In that chapter, we used stock price data of AAPL. We will load the close prices and the (always complex) dates. In fact, create a converter function just for the dates:

```
def datestr2num(s):
    return datetime.datetime.strptime(s, "%d-%m-%Y").toordinal()
dates, closes=np.loadtxt('AAPL.csv', delimiter=',',
    usecols=(1, 6), converters={1:datestr2num}, unpack=True)
```

2. Sort the names lexically with the `lexsort()` function. The data is already sorted by date, but sort it by close as well:

```
indices = np.lexsort((dates, closes))
print("Indices", indices)
print(["%s %s" % (datetime.date.fromordinal(dates[i]),
    closes[i]) for i in indices])
```

The code prints the following:

```
Indices [ 0 16  1 17 18  4  3  2  5 28 19 21 15  6 29 22 27 20  9
 7 25 26 10  8 14 11 23 12 24 13]
['2011-01-28 336.1', '2011-02-22 338.61', '2011-01-31 339.32',
'2011-02-23 342.62', '2011-02-24 342.88', '2011-02-03 343.44',
'2011-02-02 344.32', '2011-02-01 345.03', '2011-02-04 346.5',
'2011-03-10 346.67', '2011-02-25 348.16', '2011-03-01 349.31',
'2011-02-18 350.56', '2011-02-07 351.88', '2011-03-11 351.99',
'2011-03-02 352.12', '2011-03-09 352.47', '2011-02-28 353.21',
'2011-02-10 354.54', '2011-02-08 355.2', '2011-03-07 355.36',
'2011-03-08 355.76', '2011-02-11 356.85', '2011-02-09 358.16',
'2011-02-17 358.3', '2011-02-14 359.18', '2011-03-03 359.56',
'2011-02-15 359.9', '2011-03-04 360.0', '2011-02-16 363.13']
```

## What just happened?

We sorted the close prices of AAPL lexically using the NumPy `lexsort()` function. The function returned the indices corresponding with sorting the array (see `lex.py`):

```
from __future__ import print_function
import numpy as np
import datetime

def datestr2num(s):
    return datetime.datetime.strptime(s, "%d-%m-%Y").toordinal()

dates, closes=np.loadtxt('AAPL.csv', delimiter=',',
    usecols=(1, 6), converters={1:datestr2num}, unpack=True)
indices = np.lexsort((dates, closes))

print("Indices", indices)
print(["%s %s" % (datetime.date.fromordinal(int(dates[i])),
    closes[i])
    for i in indices])
```

## Have a go hero – trying a different sort order

We sorted using the dates and the close price sort order. Try a different order. Generate random numbers using the random module we learned about in the previous chapter and sort those using `lexsort()`.

## Time for action – partial sorting via selection for a fast median with the `partition()` function

The `partition()` function does partial sorting, which should be faster than full sorting, because it's less work.



For more information, please refer to [http://en.wikipedia.org/wiki/Partial\\_sorting](http://en.wikipedia.org/wiki/Partial_sorting). A common use case is getting the top 10 elements of a collection. Partial sorting doesn't guarantee the correct order within the group of top elements itself.

The first argument of the function is the array to partially sort. The second argument is an integer or a sequence of integers corresponding to indices of array elements. The `partition()` function sorts elements in those indices correctly. With one specified index, we get two partitions; with multiple indices, we get more than one partition. The sorting algorithm makes sure that elements in partitions, which are smaller than a correctly sorted element, come before this element. Otherwise, they are placed behind this element. Let's illustrate this explanation with an example. Start a Python or IPython shell and import NumPy:

```
$ ipython
```

```
In [1]: import numpy as np
```

Create an array with random elements to sort:

```
In [2]: np.random.seed(20)
```

```
In [3]: a = np.random.random_integers(0, 9, 9)
```

```
In [4]: a
```

```
Out[4]: array([3, 9, 4, 6, 7, 2, 0, 6, 8])
```

Partially sort the array by partitioning it in two roughly equal parts:

```
In [5]: np.partition(a, 4)
```

```
Out[5]: array([0, 2, 3, 4, 6, 6, 7, 9, 8])
```

We get an almost perfect sorting except for the last two elements.

### ***What just happened?***

We partially sorted a nine-element array. The sorting only guaranteed that one element in the middle at index 4 is at the correct position. This corresponds to trying to get the top five elements of the array without caring about the order within the top five group. Since the correctly sorted element is in the middle, this also gives the median of the array.

## **Complex numbers**

Complex numbers are numbers that have a real and imaginary part. As you remember from previous chapters, NumPy has special complex data types that represent complex numbers by two floating-point numbers. These numbers can be sorted using the NumPy `sort_complex()` function. This function sorts the real part first and then the imaginary part.

## Time for action – sorting complex numbers

We will create an array of complex numbers and sort it:

1. Generate five random numbers for the real part of the complex numbers and five numbers for the imaginary part. Seed the random generator to 42:

```
np.random.seed(42)
complex_numbers = np.random.random(5) + 1j *
np.random.random(5)
print("Complex numbers\n", complex_numbers)
```

2. Call the `sort_complex()` function to sort the complex numbers we generated in the previous step:

```
print("Sorted\n", np.sort_complex(complex_numbers))
```

The sorted numbers would be:

Sorted

```
[ 0.39342751+0.34955771j  0.40597665+0.77477433j
 0.41516850+0.26221878j
 0.86631422+0.74612422j  0.92293095+0.81335691j]
```

### What just happened?

We generated random complex numbers and sorted them using the `sort_complex()` function (see `sortcomplex.py`):

```
from __future__ import print_function
import numpy as np

np.random.seed(42)
complex_numbers = np.random.random(5) + 1j * np.random.random(5)
print("Complex numbers\n", complex_numbers)

print("Sorted\n", np.sort_complex(complex_numbers))
```

## Pop quiz – generating random numbers

Q1. Which NumPy module deals with random numbers?

1. Randnum
2. random
3. randomutil
4. rand

## Searching

NumPy has several functions that can search through arrays:

- ◆ The `argmax()` function gives the indices of the maximum values of an array:

```
>>> a = np.array([2, 4, 8])
>>> np.argmax(a)
2
```
- ◆ The `nanargmax()` function does the same, but ignores NaN values:

```
>>> b = np.array([np.nan, 2, 4])
>>> np.nanargmax(b)
2
```
- ◆ The `argmin()` and `nanargmin()` functions provide similar functionality but pertaining to minimum values. The `argmax()` and `nanargmax()` functions are also available as methods of the `ndarray` class.
- ◆ The `argwhere()` function searches for non-zero values and returns the corresponding indices grouped by element:

```
>>> a = np.array([2, 4, 8])
>>> np.argwhere(a <= 4)
array([[0],
       [1]])
```
- ◆ The `searchsorted()` function tells you the index in an array where a specified value belongs to maintain the sort order. It uses binary search (see <https://www.khanacademy.org/computing/computer-science/algorithms/binary-search/a/binary-search>), which is a  $O(\log n)$  algorithm. We will see this function in action shortly.
- ◆ The `extract()` function retrieves values from an array based on a condition.

### Time for action – using `searchsorted`

The `searchsorted()` function gets the index of a value in a sorted array. An example should make this clear:

1. To demonstrate, create an array with `arange()`, which of course is sorted:

```
a = np.arange(5)
```
2. Time to call the `searchsorted()` function:

```
indices = np.searchsorted(a, [-2, 7])
print("Indices", indices)
```

The indices, which should maintain the sort order:

```
Indices [0 5]
```

3. Construct the full array with the `insert()` function:

```
print("The full array", np.insert(a, indices, [-2, 7]))
```

This gives us the full array:

```
The full array [-2  0  1  2  3  4  7]
```

### ***What just happened?***

The `searchsorted()` function gave us indices 5 and 0 for 7 and -2. With these indices, we made the array `[-2, 0, 1, 2, 3, 4, 7]`, so the array remains sorted (see `sortedsearch.py`):

```
from __future__ import print_function
import numpy as np

a = np.arange(5)
indices = np.searchsorted(a, [-2, 7])
print("Indices", indices)

print("The full array", np.insert(a, indices, [-2, 7]))
```

## **Array elements extraction**

The NumPy `extract()` function allows us to extract items from an array based on a condition. This function is similar to the `where()` function we encountered in *Chapter 3, Getting Familiar with Commonly Used Functions*. The special `nonzero()` function selects non-zero elements.

### **Time for action – extracting elements from an array**

Let's extract the even elements of an array:

1. Create the array with the `arange()` function:

```
a = np.arange(7)
```

2. Create the condition that selects the even elements:

```
condition = (a % 2) == 0
```

3. Extract the even elements using our condition with the `extract()` function:

```
print("Even numbers", np.extract(condition, a))
```

This gives us the even numbers as required (`np.extract(condition, a)` is equivalent to `a[np.where(condition)[0]]`):

```
Even numbers [0 2 4 6]
```

4. Select non-zero values with the `nonzero()` function:

```
print("Non zero", np.nonzero(a))
```

This prints all the non-zero values of the array:

```
Non zero (array([1, 2, 3, 4, 5, 6]),)
```

## What just happened?

We extracted the even elements from an array using a Boolean condition with the NumPy `extract()` function (see `extracted.py`):

```
from __future__ import print_function
import numpy as np

a = np.arange(7)
condition = (a % 2) == 0
print("Even numbers", np.extract(condition, a))
print("Non zero", np.nonzero(a))
```

## Financial functions


NumPy has a number of financial functions:

- ◆ The `fv()` function calculates the so-called **future value**. The future value gives the value of a financial instrument at a future date, based on certain assumptions.
- ◆ The `pv()` function computes the present value (see <https://www.khanacademy.org/economics-finance-domain/core-finance/interest-tutorial/present-value/v/time-value-of-money>). The present value is the value of an asset today.
- ◆ The `npv()` function returns the **net present value**. The net present value is defined as the sum of all the present value cash flows.
- ◆ The `pmt()` function computes the **payment against loan** principal plus interest.

- ◆ The `irr()` function calculates the **internal rate of return**. The internal rate of return is the effective interested rate, which does not take into account inflation.
- ◆ The `mirr()` function calculates the **modified internal rate of return**. The modified internal rate of return is an improved version of the internal rate of return.
- ◆ The `nper()` function returns the **number of periodic payments**.
- ◆ The `rate()` function calculates the **rate of interest**.

## Time for action – determining the future value

The future value gives the value of a financial instrument at a future date, based on certain assumptions. The future value depends on four parameters—the interest rate, the number of periods, a periodic payment, and the present value.



Read more about future value at [http://en.wikipedia.org/wiki/Future\\_value](http://en.wikipedia.org/wiki/Future_value). The formula for future value with compound interest is as follows:

$$PV(1+r)^n$$

In the preceding formula, *PV* is the present value, *r* is the interest rate, and *n* is the number of periods.

In this section, let's take an interest rate of 3 percent, a quarterly payment of 10 for 5 years, and a present value of 1000. Call the `fV()` function with the appropriate values (negative values represent outgoing cash flow):

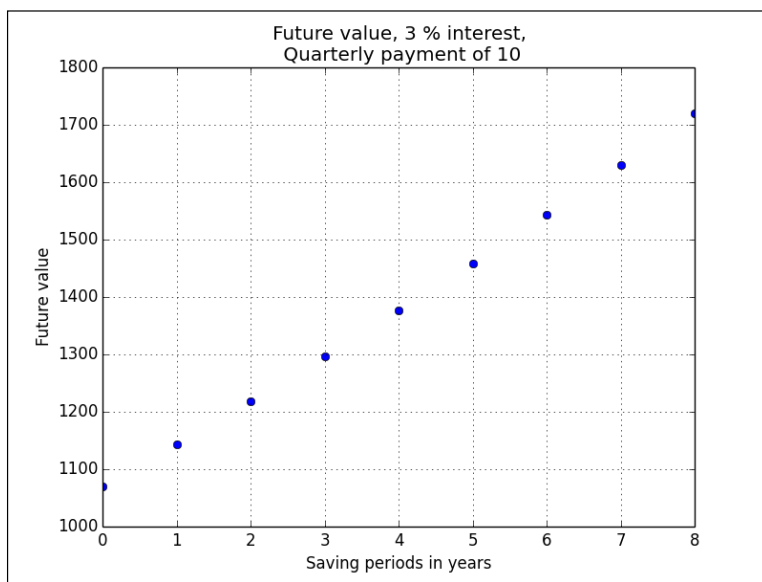
```
print("Future value", np.fv(0.03/4, 5 * 4, -10, -1000))
```

The future value is as follows:

**Future value 1376.09633204**



If we vary the number of years we save and keep the other parameters constant, we get the following plot:



### ***What just happened?***

We calculated the future value using the NumPy `fv()` function starting with a present value of 1000, an interest rate of 3 percent, and quarterly payments of 10 for 5 years. We plotted the future value for various saving periods (see `futurevalue.py`):

```
from __future__ import print_function
import numpy as np
import matplotlib.pyplot as plt

print("Future value", np.fv(0.03/4, 5 * 4, -10, -1000))

fvals = []

for i in xrange(1, 10):
    fvals.append(np.fv(.03/4, i * 4, -10, -1000))

plt.plot(range(1, 10), fvals, 'bo')
plt.title('Future value, 3 % interest,\n Quarterly payment of 10')
plt.xlabel('Saving periods in years')
plt.ylabel('Future value')
plt.grid()
plt.legend(loc='best')
plt.show()
```

## Present value

The present value is the value of an asset today. The NumPy `pv()` function can calculate the present value. This function mirrors the `fV()` function and requires the interest rate, number of periods, and the periodic payment as well, but here we start with the future value.

Read more about the present value at [http://en.wikipedia.org/wiki/Present\\_value](http://en.wikipedia.org/wiki/Present_value). It should be easy to derive the formula for the present value from the formula for the future value, if you want.

### Time for action – getting the present value

Let's reverse compute the present value with the numbers from the *Time for action – determining the future value* section:

Plug in the figures from the *Time for action – determining the future value* section:

```
print("Present value", np.pv(0.03/4, 5 * 4, -10, 1376.09633204))
```

This gives us 1000 as expected apart from a tiny numerical error. Actually, it is not an error but a representation issue. We are dealing here with outgoing cash flow, that is the reason for the negative value:

```
Present value -999.999999999
```

### What just happened?

We did the reverse computation of the *Time for action – determining the future value* section to get the present value from the future value. This was done with the NumPy `pv()` function.

## Net present value

The net present value is defined as the sum of all the present value cash flows. The NumPy `npv()` function returns the net present value of cash flows. The function requires two arguments: the rate and an array representing the cash flows.

Read more about the net present value at [http://en.wikipedia.org/wiki/Net\\_present\\_value](http://en.wikipedia.org/wiki/Net_present_value). In the formula of the net present value,  $R_t$  is the cash flow of a time period,  $r$  is the discount rate, and  $t$  is the index of the time period:

$$\sum_{t=0}^N \frac{R_t}{(1+r)^t}$$

## Time for action – calculating the net present value

We will calculate the net present value for a random generated cash flow series:

1. Generate five random values for the cash flow series. Insert -100 as the start value:

```
cashflows = np.random.randint(100, size=5)
cashflows = np.insert(cashflows, 0, -100)
print("Cashflows", cashflows)
```

The cash flows would be as follows:

```
Cashflows [-100  38  48  90  17  36]
```

2. Call the `npv()` function to calculate the net present value from the cash flow series we generated in the previous step. Use a rate of 3 percent:

```
print("Net present value", np.npv(0.03, cashflows))
```

The net present value:

```
Net present value 107.435682443
```

### What just happened?

We computed the net present value from a random generated cash flow series with the NumPy `npv()` function (see `netpresentvalue.py`):

```
from __future__ import print_function
import numpy as np

cashflows = np.random.randint(100, size=5)
cashflows = np.insert(cashflows, 0, -100)
print("Cashflows", cashflows)

print("Net present value", np.npv(0.03, cashflows))
```

## Internal rate of return

The internal rate of return is the effective interested rate, which does not take into account inflation. The NumPy `irr()` function returns the internal rate of return for a given cash flow series.

## Time for action – determining the internal rate of return

Let's reuse the cash flow series from the *Time for action – calculating the net present value* section. Call the `irr()` function with the cash flow series from the *Time for action* section:

```
print("Internal rate of return", np.irr([-100, 38, 48, 90,
    17, 36]))
```

The internal rate of return:

```
Internal rate of return 0.373420226888
```

### ***What just happened?***

We calculated the internal rate of return from the cash flow series of the *Time for action – calculating the net present value* section. The value was given by the NumPy `irr()` function.

## Periodic payments

The NumPy `pmt()` function allows you to compute periodic payments for a loan, based on an interest rate and the number of periodic payments.

## Time for action – calculating the periodic payments

Suppose you have a loan of 10 million with an interest rate of 1 percent. You have 30 years to pay the loan back. How much do you have to pay each month? Let's find out.

Call the `pmt()` function with the aforementioned values:

```
print("Payment", np.pmt(0.01/12, 12 * 30, 10000000))
```

The monthly payment:

```
Payment -32163.9520447
```

### ***What just happened?***

We calculated the monthly payment for a loan of 10 million at an annual rate of 1 percent. Given that we have 30 years to repay the loan the `pmt()` function tells us that we need to pay 32163.95 per month.

## Number of payments

The NumPy `nper()` function tells us how many periodic payments are necessary to pay off a loan. The required parameters are the interest rate of the loan, the fixed amount periodic payment, and the present value.

### Time for action – determining the number of periodic payments

Consider a loan of 9000 at a rate of 10 percent with fixed monthly payments of 100.

Find out how many payments are required with the NumPy `nper()` function:

```
print("Number of payments", np.nper(0.10/12, -100, 9000))
```

The number of payments:

```
Number of payments 167.047511801
```

### What just happened?

We determined the number of payments needed to pay off a loan of 9000 with an interest rate of 10 percent and monthly payments of 100. The number of payments returned was 167.

## Interest rate

The NumPy `rate()` function calculates the interest rate given the number of periodic payments, the payment amount or amounts, the present value, and the future value.

### Time for action – figuring out the rate

Let's take the values from the *Time for action – determining the number of periodic payments* section and reverse compute the interest rate from the other parameters.

Fill in the numbers from the previous *Time for action* section:

```
print("Interest rate", 12 * np.rate(167, -100, 9000, 0))
```

The interest rate is approximately 10 percent as expected:

```
Interest rate 0.0999756420664
```

## What just happened?

We used the NumPy `rate()` function and the values from the *Time for action – determining the number of periodic payments* section to compute the interest rate of the loan. Ignoring the rounding errors, we got the initial 10 percent we started with.

## Window functions

Window functions are mathematical functions commonly used in signal processing. Applications include spectral analysis and filter design. These functions are defined to be 0 outside a specified domain. NumPy has a number of window functions: `bartlett()`, `blackman()`, `hamming()`, `hanning()`, and `kaiser()`. You can find an example of the `hanning()` function in *Chapter 4, Convenience Functions for Your Convenience*, and *Chapter 3, Getting Familiar with Commonly Used Functions*.

### Time for action – plotting the Bartlett window

The Bartlett window is a triangular smoothing window:

$$w(n) = 1 - \left| \frac{n - \frac{N-1}{2}}{\frac{N-1}{2}} \right|$$

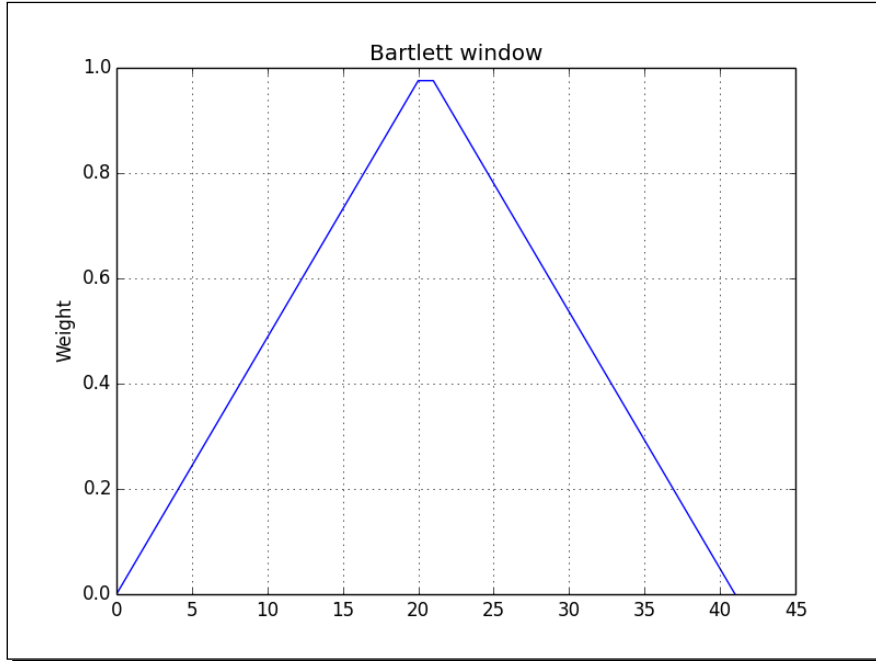
1. Call the NumPy `bartlett()` function:

```
window = np.bartlett(42)
```

2. Plotting is easy with matplotlib:

```
plt.plot(window)
plt.show()
```

The following is the Bartlett window, which is triangular, as expected:



### ***What just happened?***

We plotted the Bartlett window with the NumPy `bartlett()` function.

## **Blackman window**

The Blackman window is the sum of the following cosines:

$$w(n) = 0.42 - 0.5 \cos\left(\frac{2\pi n}{M}\right) + 0.08 \cos\left(\frac{4\pi n}{M}\right)$$

The NumPy `blackman()` function returns the Blackman window. The only parameter is the number of points `M` in the output window. If this number is 0 or less than 0, the function returns an empty array.

## Time for action – smoothing stock prices with the Blackman window

Let's smooth the close prices from the small AAPL stock prices data file:

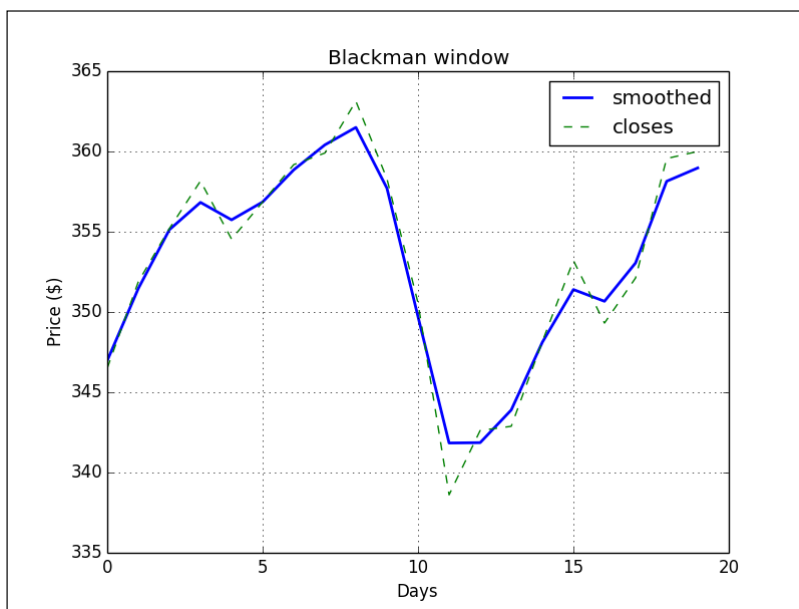
1. Load the data into a NumPy array. Call the NumPy `blackman()` function to form a window, and then use this window to smooth the price signal:

```
closes=np.loadtxt('AAPL.csv', delimiter=',', usecols=(6, ),
converters={1:datestr2num}, unpack=True)
N = 5
window = np.blackman(N)
smoothed = np.convolve(window/window.sum(),
    closes, mode='same')
```

2. Plot the smoothed prices with matplotlib. In this example, we will omit the first five data points and the last five data points. The reason for this is that there is a strong boundary effect:

```
plt.plot(smoothed[N:-N], lw=2, label="smoothed")
plt.plot(closes[N:-N], label="closes")
plt.legend(loc='best')
plt.show()
```

The closing prices of AAPL smoothed with the Blackman window should appear as follows:





## What just happened?

We plotted the closing price of AAPL from our sample data file that was smoothed using the Blackman window with the NumPy `blackman()` function (see `plot_blackman.py`):

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.dates import datestr2num

closes=np.loadtxt('AAPL.csv', delimiter=',', usecols=(6,),
converters={1:datestr2num}, unpack=True)
N = 5
window = np.blackman(N)
smoothed = np.convolve(window/window.sum(), closes, mode='same')
plt.plot(smoothed[N:-N], lw=2, label="smoothed")
plt.plot(closes[N:-N], '--', label="closes")
plt.title('Blackman window')
plt.xlabel('Days')
plt.ylabel('Price ($)')
plt.grid()
plt.legend(loc='best')
plt.show()
```

## Hamming window

The Hamming window is formed by a weighted cosine. The formula is as follows:

$$w(n) = 0.54 + 0.46 \cos\left(\frac{2\pi n}{M-1}\right) \quad 0 \leq n \leq M-1$$

The NumPy `hamming()` function returns the Hamming window. The only parameter is the number of points  $M$  in the output window. If this number is 0 or less than 0, an empty array is returned.

## Time for action – plotting the Hamming window

Let's plot the Hamming window:

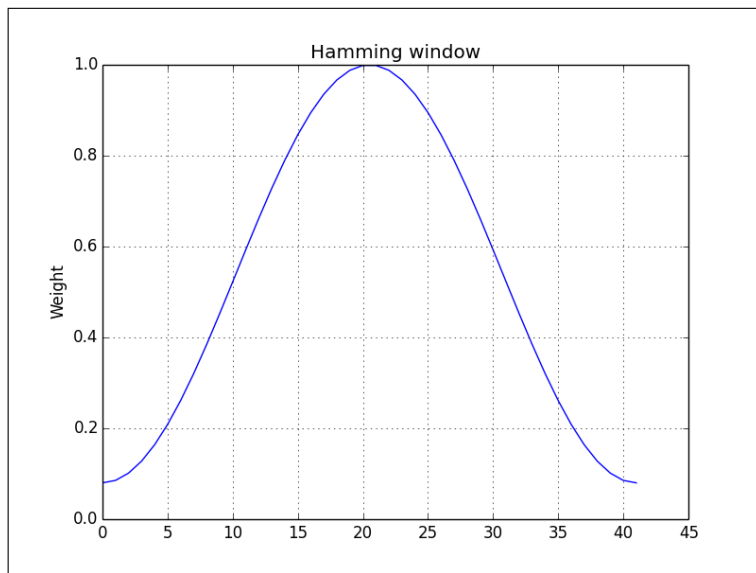
1. Call the NumPy `hamming()` function:

```
window = np.hamming(42)
```

2. Plot the window with `matplotlib`:

```
plt.plot(window)
plt.show()
```

The Hamming window plot appears as follows:



### ***What just happened?***

We plotted the Hamming window with the NumPy `hamming()` function.

## **Kaiser window**

The **Kaiser window** is formed by the **Bessel function**.



Bessel functions are solutions of the Bessel differential equations (see [http://en.wikipedia.org/wiki/Bessel\\_function](http://en.wikipedia.org/wiki/Bessel_function)).

The formula is as follows:

$$w(n) = I_0 \left( \beta \sqrt{1 - \frac{4n^2}{(M-1)^2}} \right) / I_0(\beta)$$

Here  $I_0$  is the zero order Bessel function. The NumPy `kaiser()` function returns the **Kaiser window**. The first parameter is the number of points in the output window. If this number is 0 or less than 0, the function returns an empty array. The second parameter is the beta.

## Time for action – plotting the Kaiser window

Let's plot the Kaiser window:

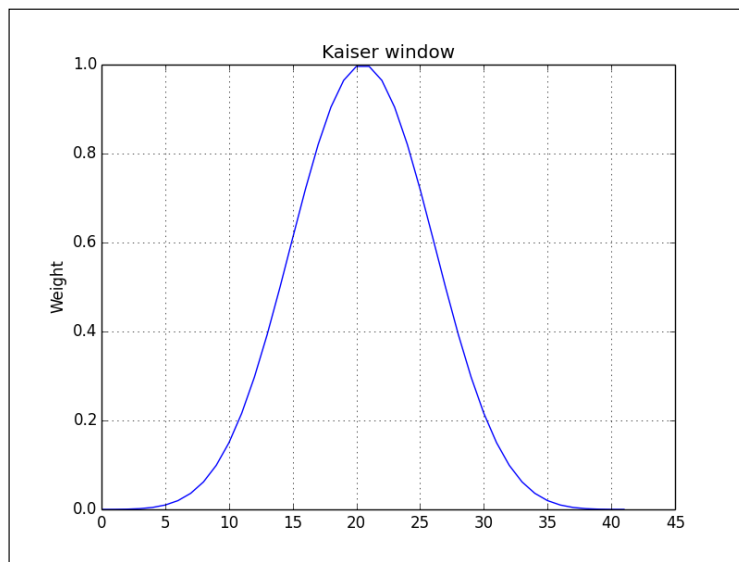
1. Call the NumPy `kaiser()` function:

```
window = np.kaiser(42, 14)
```

2. Plot the window with matplotlib:

```
plt.plot(window)  
plt.show()
```

The Kaiser window appears as follows:



### ***What just happened?***

We plotted the Kaiser window with the NumPy `kaiser()` function.

## Special mathematical functions

We will end this chapter with some special mathematical functions. The modified Bessel function of the first kind 0th order is represented in NumPy by `i0()`. The `sinc` function is represented in NumPy by a function with the same name, and there is also a two-dimensional version of this function. **Sinc** is a trigonometric function; for more details, see [http://en.wikipedia.org/wiki/Sinc\\_function](http://en.wikipedia.org/wiki/Sinc_function). The `sinc()` function has two definitions.

The NumPy `sinc()` function complies with the following definition:

$$\frac{\sin(\pi x)}{\pi x}$$

## Time for action – plotting the modified Bessel function

Let's see what the modified Bessel function of the first kind 0th order looks like:

1. Compute evenly spaced values with the NumPy `linspace()` function:

```
x = np.linspace(0, 4, 100)
```

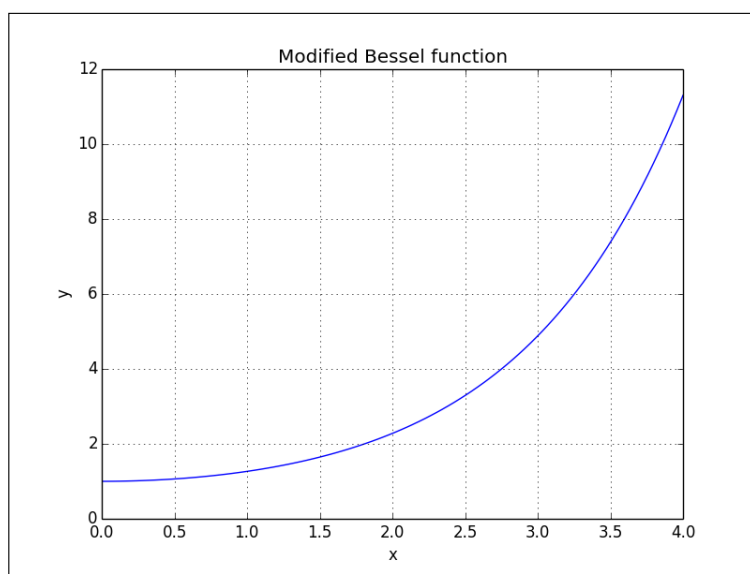
2. Call the NumPy `i0()` function:

```
vals = np.i0(x)
```

3. Plot the modified Bessel function with matplotlib:

```
plt.plot(x, vals)  
plt.show()
```

The modified Bessel function will have the following output:



### What just happened?

We plotted the modified Bessel function of the first kind 0th order with the NumPy `i0()` function.

## sinc

The `sinc()` function is widely used in mathematics and signal processing. NumPy has a function with the same name. A two-dimensional function exists as well.

### Time for action – plotting the sinc function

We will plot the `sinc()` function:

1. Compute evenly spaced values with the NumPy `linspace()` function:

```
x = np.linspace(0, 4, 100)
```

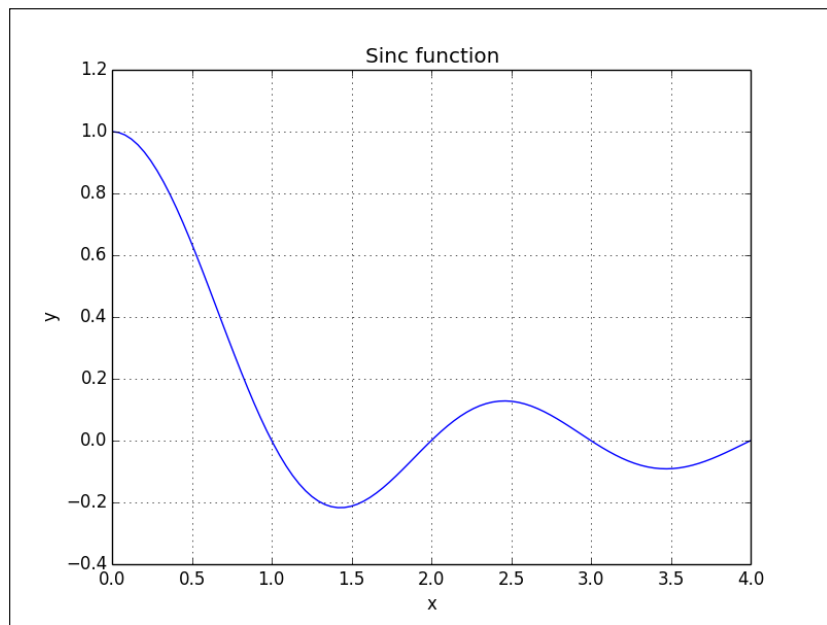
2. Call the NumPy `sinc()` function:

```
vals = np.sinc(x)
```

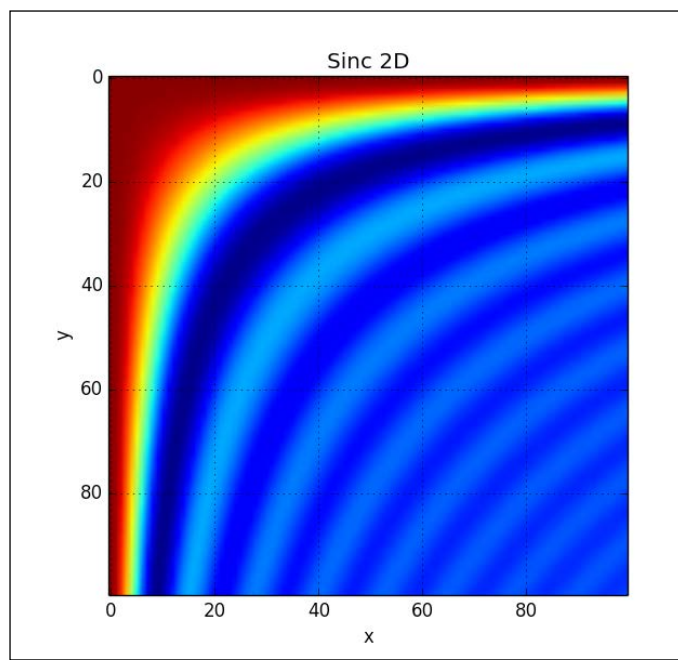
3. Plot the `sinc()` function with `matplotlib`:

```
plt.plot(x, vals)  
plt.show()
```

The `sinc()` function will have the following output:



The `sinc2d()` function requires a two-dimensional array. We can create it with the `outer()` function, resulting in this plot (code is in the following section):



### ***What just happened?***

We plotted the well-known `sinc` function with the NumPy `sinc()` function (see `plot_sinc.py`):

```
import numpy as np
import matplotlib.pyplot as plt

x = np.linspace(0, 4, 100)
vals = np.sinc(x)

plt.plot(x, vals)
plt.title('Sinc function')
plt.xlabel('x')
plt.ylabel('y')
plt.grid()
plt.show()
```

We did the same for two dimensions (see `sinc2d.py`):

```
import numpy as np
import matplotlib.pyplot as plt

x = np.linspace(0, 4, 100)
xx = np.outer(x, x)
vals = np.sinc(xx)

plt.imshow(vals)
plt.title('Sinc 2D')
plt.xlabel('x')
plt.ylabel('y')
plt.grid()
plt.show()
```

## Summary

This was a special chapter covering more specialized NumPy topics. We covered sorting and searching, special functions, financial utilities, and window functions.

The next chapter is about the very important subject of testing.

# 8

## Assuring Quality with Testing

*Some programmers test only in production. If you are not one of them, then you're probably familiar with the concept of unit testing. Unit tests are automated tests written by a programmer to test his or her code. These tests could, for example, test a function or part of a function in isolation. Each test covers only a small unit of code. The benefits are increased confidence in the quality of the code, reproducible tests, and, as a side effect, clearer code.*

*Python has good support for unit testing. Additionally, NumPy adds the `numpy.testing` package to that for NumPy code unit testing.*

**Test-driven development (TDD)** is one of the most important things that happened to software development. TDD focuses a lot on automated unit testing. The goal is to test automatically the code as much as possible. The next time we change the code, we can run the tests and catch potential regressions. In other words, any functionality already present will still work.

The topics in this chapter include the following:

- ◆ Unit testing
- ◆ Asserts
- ◆ Floating-point precision



## Assert functions

Unit tests usually use functions, which assert something as part of the test. When doing numerical calculations, often we have the fundamental issue of trying to compare floating-point numbers that are almost equal. For integers, comparison is a trivial operation, but for floating-point numbers it is not, because of the inexact representation by computers. The NumPy `testing` package has a number of utility functions that test whether a precondition is true or not, taking into account the problem of floating-point comparisons. The following table shows the different utility functions:

Function	Description
<code>assert_almost_equal()</code>	This function raises an exception if two numbers are not equal up to a specified precision
<code>assert_approx_equal()</code>	This function raises an exception if two numbers are not equal up to a certain significance
<code>assert_array_almost_equal()</code>	This function raises an exception if two arrays are not equal up to a specified precision
<code>assert_array_equal()</code>	This function raises an exception if two arrays are not equal.
<code>assert_array_less()</code>	This function raises an exception if two arrays do not have the same shape, and the elements of the first array are strictly less than the elements of the second array
<code>assert_equal()</code>	This function raises an exception if two objects are not equal
<code>assert_raises()</code>	This function fails if a specified exception is not raised by a callable invoked with defined arguments
<code>assert_warns()</code>	This function fails if a specified warning is not thrown
<code>assert_string_equal()</code>	This function asserts that two strings are equal
<code>assert_allclose()</code>	This function raise an assertion if two objects are not equal up to desired tolerance

### Time for action – asserting almost equal

Imagine that you have two numbers that are almost equal. Let's use the `assert_almost_equal()` function to check whether they are equal:

1. Call the function with low precision (up to 7 decimal places):

```
print("Decimal 6", np.testing.assert_almost_equal(0.123456789,
0.123456780, decimal=7))
```

Note that no exception is raised, as you can see in the following result:

```
Decimal 6 None
```

2. Call the function with higher precision (up to 8 decimal places):

```
print("Decimal 7", np.testing.assert_almost_equal(0.123456789,
0.123456780, decimal=8))
```

The result is as follows:

```
Decimal 7
Traceback (most recent call last):
...
  raise AssertionError(msg)
AssertionError:
Arrays are not almost equal
ACTUAL: 0.123456789
DESIRED: 0.12345678
```

### ***What just happened?***

We used the `assert_almost_equal()` function from the NumPy testing package to check whether `0.123456789` and `0.123456780` are equal for different decimal precisions.

### **Pop quiz – specifying decimal precision**

Q1. Which parameter of the `assert_almost_equal()` function specifies the decimal precision?

1. decimal
2. precision
3. tolerance
4. significant

### **Approximately equal arrays**

The `assert_approx_equal()` function raises an exception if two numbers are not equal up to a certain number of significant digits. The function raises an exception triggered by the following condition:

```
abs(actual - expected) >= 10**-(significant - 1)
```

## Time for action – asserting approximately equal

Let's take the numbers from the previous *Time for action* section and let the `assert_approx_equal()` function work on them:

1. Call the function with low significance:

```
print("Significance 8",
      np.testing.assert_approx_equal
      (0.123456789, 0.123456780, significant=8))
```

The result is as follows:

```
Significance 8 None
```

2. Call the function with high significance:

```
print("Significance 9",
      np.testing.assert_approx_equal
      (0.123456789, 0.123456780, significant=9))
```

The function raises an `AssertionError`:

```
Significance 9
Traceback (most recent call last):
...
    raise AssertionError(msg)
AssertionError:
Items are not equal to 9 significant digits:
ACTUAL: 0.123456789
DESIRED: 0.12345678
```

### ***What just happened?***

We used the `assert_approx_equal()` function from the NumPy `testing` package to check whether `0.123456789` and `0.123456780` are equal for different decimal precisions.

### **Almost equal arrays**

The `assert_array_almost_equal()` function raises an exception if two arrays are not equal up to a specified precision. The function checks whether the two arrays have the same shape. Then, the values of the arrays are compared element by element with the following:

```
|expected - actual| < 0.5 * 10-decimal
```

## Time for action – asserting arrays almost equal

Let's form arrays with the values from the previous *Time for action* section by adding a 0 to each array:

1. Call the function with lower precision:

```
print("Decimal 8", np.testing.assert_array_almost_equal([0,
    0.123456789], [0, 0.123456780], decimal=8))
```

The result is as follows:

```
Decimal 8 None
```

2. Call the function with higher precision:

```
print("Decimal 9", np.testing.assert_array_almost_equal([0,
    0.123456789], [0, 0.123456780], decimal=9))
```

The test raises an `AssertionError`:

```
Decimal 9
```

```
Traceback (most recent call last):
```

```
...
```

```
assert_array_compare
```

```
    raise AssertionError(msg)
```

```
AssertionError:
```

```
Arrays are not almost equal
```

```
(mismatch 50.0%)
```

```
x: array([ 0.          ,  0.12345679])
```

```
y: array([ 0.          ,  0.12345678])
```

### What just happened?

We compared two arrays with the NumPy `array_almost_equal()` function.

## Have a go hero – comparing arrays with different shapes

Use the NumPy `array_almost_equal()` function to compare two arrays with different shapes.

## Equal arrays

The `assert_array_equal()` function raises an exception if two arrays are not equal. The shapes of the arrays have to be equal and the elements of each array must be equal. NaNs are allowed in the arrays. Alternatively, arrays can be compared with the `array_allclose()` function. This function has the parameters **absolute tolerance (atol)** and **relative tolerance (rtol)**. For two arrays `a` and `b`, these parameters satisfy the following equation:

$$|a - b| \leq (atol + rtol * |b|)$$

### Time for action – comparing arrays

Let's compare two arrays with the functions we just mentioned. We will reuse the arrays from the previous *Time for action* section and add a NaN to them:

1. Call the `array_allclose()` function:

```
print("Pass", np.testing.assert_allclose([0, 0.123456789,
    np.nan], [0, 0.123456780, np.nan], rtol=1e-7, atol=0))
```

The result is as follows:

```
Pass None
```

2. Call the `array_equal()` function:

```
print("Fail", np.testing.assert_array_equal([0, 0.123456789,
    np.nan], [0, 0.123456780, np.nan]))
```

The test fails with an `AssertionError`:

```
Fail
```

```
Traceback (most recent call last):
```

```
...
assert_array_compare
    raise AssertionError(msg)
```

```
AssertionError:
```

```
Arrays are not equal
```

```
(mismatch 50.0%)
```

```
x: array([ 0.          ,  0.12345679,   nan])
y: array([ 0.          ,  0.12345678,   nan])
```

## What just happened?

We compared two arrays with the `array_allclose()` function and the `array_equal()` function.

## Ordering arrays

The `assert_array_less()` function raises an exception if two arrays do not have the same shape, and the elements of the first array are strictly less than the elements of the second array.

### Time for action – checking the array order

Let's check whether one array is strictly greater than another array:

1. Call the `assert_array_less()` function with two strictly ordered arrays:

```
print("Pass", np.testing.assert_array_less([0, 0.123456789,
                                             np.nan], [1, 0.23456780, np.nan]))
```

The result is as follows:

```
Pass None
```

2. Call the `assert_array_less()` function:

```
print("Fail", np.testing.assert_array_less([0, 0.123456789,
                                             np.nan], [0, 0.123456780, np.nan]))
```

The test raises an exception:

```
Fail
```

```
Traceback (most recent call last):
```

```
...
```

```
    raise AssertionError(msg)
```

```
AssertionError:
```

```
Arrays are not less-ordered
```

```
(mismatch 100.0%)
```

```
x: array([ 0.          ,  0.12345679,          nan])
```

```
y: array([ 0.          ,  0.12345678,          nan])
```

## ***What just happened?***

We checked the ordering of two arrays with the `assert_array_less()` function.

## **Object comparison**

The `assert_equal()` function raises an exception if two objects are not equal. The objects do not have to be NumPy arrays—they can also be lists, tuples, or dictionaries.

### **Time for action – comparing objects**

Suppose you need to compare two tuples. We can use the `assert_equal()` function to do that.

Call the `assert_equal()` function:

```
print("Equal?", np.testing.assert_equal((1, 2), (1, 3)))
```

The call raises an error because the items are not equal:

```
Equal?
Traceback (most recent call last):
...
    raise AssertionError(msg)
AssertionError:
Items are not equal:
item=1

ACTUAL: 2
DESIRED: 3
```

## ***What just happened?***

We compared two tuples with the `assert_equal()` function—an exception was raised because the tuples were not equal to each other.

## **String comparison**

The `assert_string_equal()` function asserts that two strings are equal. If the test fails, the function throws an exception and shows the difference between the strings. The case of the string characters matters.

## Time for action – comparing strings

Let's compare strings. Both strings are the word "NumPy":

1. Call the `assert_string_equal()` function to compare a string with itself. This test, of course, should pass:

```
print("Pass", np.testing.assert_string_equal("NumPy", "NumPy"))
```

The test passes:

```
Pass None
```

2. Call the `assert_string_equal()` function to compare a string with another string with the same letters, but different casing. This test should throw an exception:

```
print("Fail", np.testing.assert_string_equal("NumPy", "Numpy"))
```

The test raises an error:

```
Fail
```

```
Traceback (most recent call last):
```

```
...
```

```
    raise AssertionError(msg)
```

```
AssertionError: Differences in strings:
```

```
- NumPy?      ^
```

```
+ Numpy?      ^
```

### What just happened?

We compared two strings with the `assert_string_equal()` function. The test threw an exception when the casing did not match.

## Floating-point comparisons

The representation of floating-point numbers in computers is not exact. This leads to issues when comparing floating-point numbers. The `assert_array_almost_equal_nulp()` and `assert_array_max_ulp()` NumPy functions provide consistent floating-point comparisons. **Unit of Least Precision (ULP)** of floating-point numbers, according to the IEEE 754 specification, a half ULP precision is required for elementary arithmetic operations. You can compare this to a ruler. A metric system ruler usually has ticks for millimeters, but beyond that you can only estimate half millimeters.



**Machine epsilon** is the largest relative rounding error in floating-point arithmetic. Machine epsilon is equal to ULP relative to 1. The NumPy `finfo()` function allows us to determine the machine epsilon. The Python standard library also can give you the machine epsilon value. The value should be the same as that given by NumPy.

## Time for action – comparing with `assert_array_almost_equal_nulp`

Let's see the `assert_array_almost_equal_nulp()` function in action:

1. Determine the machine epsilon with the `finfo()` function:

```
eps = np.finfo(float).eps
print("EPS", eps)
```

The epsilon would be as follows:

```
EPS 2.22044604925e-16
```

2. Compare `1.0` with `1 + epsilon` using the `assert_almost_equal_nulp()` function. Do the same for `1 + 2 * epsilon`:

```
print("1",
      np.testing.assert_array_almost_equal_nulp(1.0, 1.0 + eps))
print("2",
      np.testing.assert_array_almost_equal_nulp(1.0, 1.0 + 2 * eps))
```

The result is as follows:

```
1 None
```

```
2
```

```
Traceback (most recent call last):
```

```
...
```

```
assert_array_almost_equal_nulp
```

```
    raise AssertionError(msg)
```

```
AssertionError: X and Y are not equal to 1 ULP (max is 2)
```

### What just happened?

We determined the machine epsilon with the `finfo()` function. We then compared `1.0` with `1 + epsilon` with the `assert_almost_equal_nulp()` function. This test passed however, adding another epsilon resulted in an exception.

## Comparison of floats with more ULPs

The `assert_array_max_ulp()` function allows you to specify an upper bound for the number of ULPs you would allow. The `maxulp` parameter accepts an integer value for the limit. The value of this parameter is 1 by default.

### Time for action – comparing using maxulp of 2

Let's do the same comparisons as in the previous *Time for action* section, but specify a `maxulp` of 2 when necessary:

1. Determine the machine epsilon with the `finfo()` function:

```
eps = np.finfo(float).eps
print("EPS", eps)
```

The epsilon would be as follows:

```
EPS 2.22044604925e-16
```

2. Do the comparisons as done in the previous *Time for action* section, but use the `assert_array_max_ulp()` function with the appropriate `maxulp` value:

```
print("1", np.testing.assert_array_max_ulp(1.0, 1.0 + eps))
print("2", np.testing.assert_array_max_ulp(1.0, 1 + 2 * eps,
      maxulp=2))
```

The output is as follows:

```
1 1.0
2 2.0
```

### What just happened?

We compared the same values as the previous *Time for action* section, but specified a `maxulp` of 2 in the second comparison. Using the `assert_array_max_ulp()` function with the appropriate `maxulp` value, these tests passed with a return value of the number of ULPs.

## Unit tests

Unit tests are automated tests, which test a small piece of code, usually a function or method. Python has the `PyUnit` API for unit testing. As NumPy users, we can make use of the assert functions we saw in action before.

## Time for action – writing a unit test

We will write tests for a simple factorial function. The tests will check for the so-called happy path and abnormal conditions.

**1.** Start by writing the factorial function:

```
import numpy as np
import unittest

def factorial(n):
    if n == 0:
        return 1

    if n < 0:
        raise ValueError, "Unexpected negative value"

    return np.arange(1, n+1).cumprod()
```

The code uses the `arange()` and `cumprod()` functions to create arrays and calculate the cumulative product, but we added a few checks for boundary conditions.

**2.** Now we will write the unit test. Let's write a class that will contain the unit tests. It extends the `TestCase` class from the `unittest` module, which is part of standard Python. Test for calling the factorial function with the following three attributes:

- ❑ a positive number, the happy path
- ❑ boundary condition 0
- ❑ negative numbers, which should result in an error

```
class FactorialTest(unittest.TestCase):
    def test_factorial(self):
        #Test for the factorial of 3 that should pass.
        self.assertEqual(6, factorial(3)[-1])
        np.testing.assert_equal(np.array([1, 2, 6]),
            factorial(3))

    def test_zero(self):
        #Test for the factorial of 0 that should pass.
        self.assertEqual(1, factorial(0))

    def test_negative(self):
        #Test for the factorial of negative numbers that
        should fail.
        # It should throw a ValueError, but we expect
        IndexError
        self.assertRaises(IndexError, factorial(-10))
```

We rigged one of the tests to fail, as you can see in the following output:

```
$ python unit_test.py
.E.
=====
====
ERROR: test_negative (__main__.FactorialTest)
-----
----
Traceback (most recent call last):
  File "unit_test.py", line 26, in test_negative
    self.assertRaises(IndexError, factorial(-10))
  File "unit_test.py", line 9, in factorial
    raise ValueError, "Unexpected negative value"
ValueError: Unexpected negative value
-----
----
Ran 3 tests in 0.003s

FAILED (errors=1)
```

### ***What just happened?***

We made some happy path tests for the factorial function code. We let the boundary condition test fail on purpose (see `unit_test.py`):

```
import numpy as np
import unittest

def factorial(n):
    if n == 0:
        return 1

    if n < 0:
        raise ValueError, "Unexpected negative value"

    return np.arange(1, n+1).cumprod()

class FactorialTest(unittest.TestCase):
    def test_factorial(self):
        #Test for the factorial of 3 that should pass.
        self.assertEqual(6, factorial(3)[-1])
        np.testing.assert_equal(np.array([1, 2, 6]), factorial(3))
```

```
def test_zero(self):
    #Test for the factorial of 0 that should pass.
    self.assertEqual(1, factorial(0))

def test_negative(self):
    #Test for the factorial of negative numbers that should fail.
    # It should throw a ValueError, but we expect IndexError
    self.assertRaises(IndexError, factorial(-10))

if __name__ == '__main__':
    unittest.main()
```

## Nose test decorators

A nose is an organ above the mouth that is used by humans and animals to breathe and smell. It is also a Python framework that makes (unit) testing easier. Nose helps you organize tests. According to the nose documentation:

*"Any python source file, directory or package that matches the testMatch regular expression (by default: (? : ^ | [b \_ . - ] ) [Tt] est) will be collected as a test."*

Nose makes extensive use of decorators. Python decorators are annotations that indicate something about a method or a function (see <http://thecodship.com/patterns/guide-to-python-function-decorators/>). The `numpy.testing` module has a number of decorators. The following table shows the different decorators in the `numpy.testing` module:

Decorator	Description
<code>numpy.testing.decorators.deprecated</code>	This function filters deprecation warnings when running tests
<code>numpy.testing.decorators.knownfailureif</code>	This function raises <code>KnownFailureTest</code> exception based on a condition
<code>numpy.testing.decorators.setastest</code>	This decorator marks a function as being a test or not being a test
<code>numpy.testing.decorators.skipif</code>	This function raises a <code>SkipTest</code> exception based on a condition
<code>numpy.testing.decorators.slow</code>	This function labels test functions or methods as slow

Additionally, we can call the `decorate_methods()` function to apply decorators on methods of a class matching a regular expression or a string.

## Time for action – decorating tests

We will apply the `@setastest` decorator directly to test functions. Then we will apply the same decorator to a method to disable it. Also, we will skip one of the tests and fail another. First, install `nose` in case you don't have it yet.

1. Install `nose` with `setuptools`:

```
$ [sudo] easy_install nose
```

Or pip:

```
$ [sudo] pip install nose
```

2. Apply one function as being a test and another as not being a test:

```
@setastest(False)
def test_false():
    pass
```

```
@setastest(True)
def test_true():
    pass
```

3. Skip tests with the `@skipif` decorator. Let's use a condition that always leads to a test being skipped:

```
@skipif(True)
def test_skip():
    pass
```

4. Add a test function that always passes. Then, decorate it with the `@knownfailureif` decorator so that the test always fails:

```
@knownfailureif(True)
def test_alwaysfail():
    pass
```

5. Define some test classes with methods that normally should be executed by `nose`:

```
class TestClass():
    def test_true2(self):
        pass
```

```
class TestClass2():
    def test_false2(self):
        pass
```

6. Let's disable the second test method from the previous step:

```
decorate_methods(TestClass2, setastest(False), 'test_false2')
```

7. Run the tests with the following command:

```
$ nosetests -v decorator_setastest.py
decorator_setastest.TestClass.test_true2 ... ok
decorator_setastest.test_true ... ok
decorator_test.test_skip ... SKIP: Skipping test: test_skipTest
skipped due to test condition
decorator_test.test_alwaysfail ... ERROR

=====
====
ERROR: decorator_test.test_alwaysfail
-----
----
Traceback (most recent call last):
  File ".../nose/case.py", line 197, in runTest
    self.test(*self.arg)
  File .../numpy/testing/decorators.py", line 213, in knownfailer
    raise KnownFailureTest(msg)
KnownFailureTest: Test skipped due to known failure

-----
----
Ran 4 tests in 0.001s

FAILED (SKIP=1, errors=1)
```

### ***What just happened?***

We decorated some functions and methods as not being tests so that they were ignored by nose. We skipped one test and failed another too. We did this by applying decorators directly and with the `decorate_methods()` function (see `decorator_test.py`):

```
from numpy.testing.decorators import setastest
from numpy.testing.decorators import skipif
```

---

```
from numpy.testing.decorators import knownfailureif
from numpy.testing import decorate_methods

@setastest(False)
def test_false():
    pass

@setastest(True)
def test_true():
    pass

@skipif(True)
def test_skip():
    pass

@knownfailureif(True)
def test_alwaysfail():
    pass

class TestClass():
    def test_true2(self):
        pass

class TestClass2():
    def test_false2(self):
        pass

decorate_methods(TestClass2, setastest(False), 'test_false2')
```

## Docstrings

**Doctests** are strings embedded in Python code that resemble interactive sessions. These strings can be used to test certain assumptions or just to provide examples. The `numpy.testing` module has a function to run these tests.



## Time for action – executing doctests

Let's write a simple example that is supposed to calculate the well-known factorial, but doesn't cover all of the possible boundary conditions. In other words, some tests will fail.

1. The `docstring` will look like text you would see in a Python shell (including a prompt). Rig one of the tests to fail, just to see what will happen:

```
"""
Test for the factorial of 3 that should pass.
>>> factorial(3)
6
Test for the factorial of 0 that should fail.
>>> factorial(0)
1
"""
```

2. Write the following line of NumPy code:

```
return np.arange(1, n+1).cumprod() [-1]
```

We want this code to fail from time to time for demonstration purposes.

3. Run the doctest by calling the `rundocs()` function of the `numpy.testing` module, for instance, in the Python shell:

```
>>> from numpy.testing import rundocs
>>> rundocs('docstringtest.py')
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File ".../numpy/testing/utils.py", line 998, in rundocs
    raise AssertionError("Some doctests failed:\n%s" % "\n".
join(msg))
AssertionError: Some doctests failed:
*****
File "docstringtest.py", line 10, in docstringtest.factorial
Failed example:
    factorial(0)
```

```
Exception raised:
  Traceback (most recent call last):
    File ".../doctest.py", line 1254, in __run
      compileflags, 1) in test.globs
    File "<doctest docstringtest.factorial[1]>", line 1, in
  <module>
    factorial(0)
    File "docstringtest.py", line 13, in factorial
      return np.arange(1, n+1).cumprod() [-1]
  IndexError: index -1 is out of bounds for axis 0 with size 0
```

## What just happened?

We wrote a docstring test, which didn't take into account 0 and negative numbers. We ran the test with the `rundocs()` function from the `numpy.testing` module and got an index error as a result (see `docstringtest.py`):

```
import numpy as np

def factorial(n):
    """
    Test for the factorial of 3 that should pass.
    >>> factorial(3)
    6

    Test for the factorial of 0 that should fail.
    >>> factorial(0)
    1
    """
    return np.arange(1, n+1).cumprod() [-1]
```

## Summary

You learned about testing and NumPy testing utilities in this chapter. We covered unit testing, docstring tests, assert functions, and floating-point precision. Most of the NumPy assert functions take care of the complexities of floating-point numbers. We demonstrated NumPy decorators that can be used by nose. Decorators make testing easier and document the developer intention.

The topic of the next chapter is matplotlib—the Python scientific visualization and graphing open source library.



# 9

## Plotting with matplotlib

*matplotlib is a very useful Python plotting library. It integrates nicely with NumPy but is a separate open source project. You can find a gallery of beautiful examples at <http://matplotlib.org/gallery.html>.*

*matplotlib also has utility functions to download and manipulate data from Yahoo Finance. We will see several examples of stock charts.*

This chapter features extended coverage of the following topics:

- ◆ Simple plots
- ◆ Subplots
- ◆ Histograms
- ◆ Plot customization
- ◆ Three-dimensional plots
- ◆ Contour plots
- ◆ Animation
- ◆ Logplots

### Simple plots

The `matplotlib.pyplot` package contains functionality for simple plots. It is important to remember that each subsequent function call changes the state of the current plot. Eventually, we will want to either save the plot in a file or display it with the `show()` function. However, if we are in IPython running on a Qt or Wx backend, the figure updates interactively without waiting for the `show()` function. This is comparable to the way text output is printed on the fly.

## Time for action – plotting a polynomial function

To illustrate how plotting works, let's display some polynomial graphs. We will use the NumPy polynomial function `poly1d()` to create a polynomial.

1. Take the standard input values as polynomial coefficients. Use the NumPy `poly1d()` function to create a polynomial:  

```
func = np.poly1d(np.array([1, 2, 3, 4]).astype(float))
```
2. Create the `x` values with the NumPy the `linspace()` function. Use the range -10 to 10 and create 30 even spaced values:  

```
x = np.linspace(-10, 10, 30)
```
3. Calculate the polynomial values using the polynomial we created in the first step:  

```
y = func(x)
```
4. Call the `plot()` function; this does not immediately display the graph:  

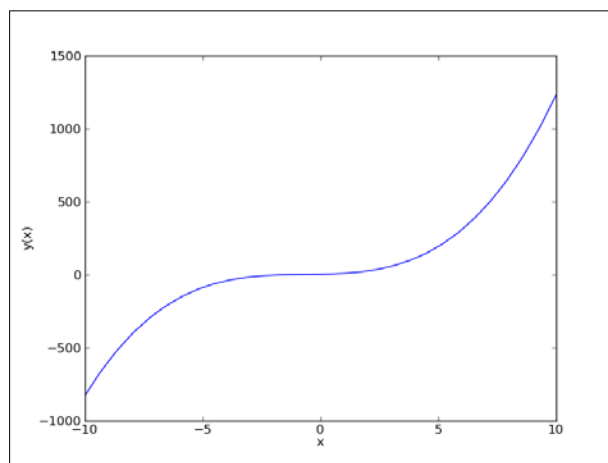
```
plt.plot(x, y)
```
5. Add a label to the `x` axis with the `xlabel()` function:  

```
plt.xlabel('x')
```
6. Add a label to the `y` axis with the `ylabel()` function:  

```
plt.ylabel('y(x)')
```
7. Call the `show()` function to display the graph:  

```
plt.show()
```

The following is a plot with polynomial coefficients 1, 2, 3, and 4:



## What just happened?

We displayed a polynomial graph on our screen. We added labels to the x and y axes (see `polyplot.py`):

```
import numpy as np
import matplotlib.pyplot as plt

func = np.poly1d(np.array([1, 2, 3, 4]).astype(float))
x = np.linspace(-10, 10, 30)
y = func(x)

plt.plot(x, y)
plt.xlabel('x')
plt.ylabel('y(x)')
plt.show()
```

## Pop quiz – the `plot()` function

Q1. What does the `plot()` function do?

1. It displays two-dimensional plots on screen.
2. It saves an image of a two-dimensional plot in a file.
3. It does both a and b.
4. It does neither a, b, or c.

## Plot format string

The `plot()` function accepts an unlimited number of arguments. In the previous section, we gave it two arrays as arguments. We could also specify the line color and style with an optional format string. By default, it is a solid blue line denoted as `b-`, but you can specify a different color and style, such as red dashes.

## Time for action – plotting a polynomial and its derivatives

Let's plot a polynomial and its first-order derivative using the `deriv()` function with `m` as 1. We already did the first part in the previous *Time for action* section. We want two different line styles to discern what is what.

1. Create and differentiate the polynomial:

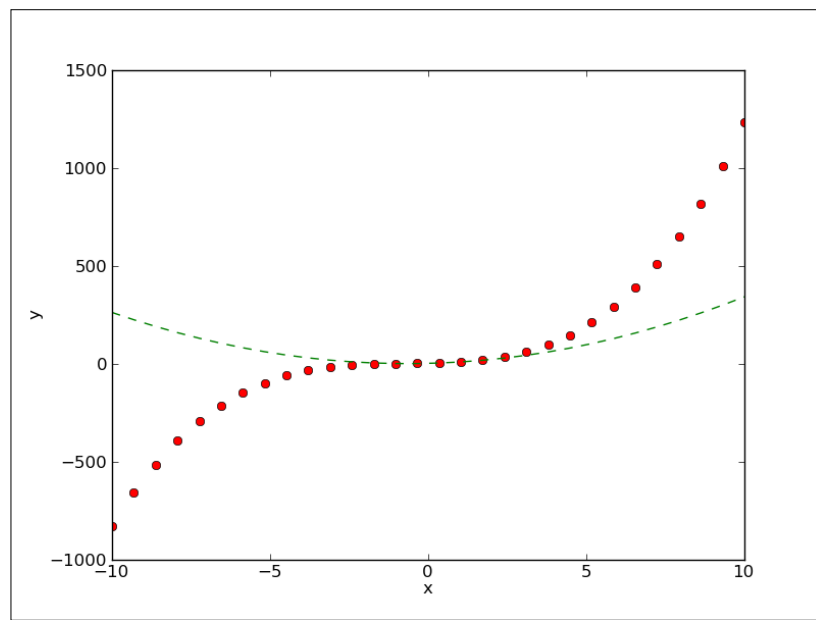
```
func = np.poly1d(np.array([1, 2, 3, 4]).astype(float))
func1 = func.deriv(m=1)
```

```
x = np.linspace(-10, 10, 30)
y = func(x)
y1 = func1(x)
```

2. Plot the polynomial and its derivative in two styles: red circles and green dashes. You cannot see the colors in a print copy of this book, so you will have to try the code out for yourself:

```
plt.plot(x, y, 'ro', x, y1, 'g--')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

The graph with polynomial coefficients 1, 2, 3, and 4 is as follows:



### ***What just happened?***

We plotted a polynomial and its derivative using two different line styles and one call of the `plot()` function (see `polyplot2.py`):

```
import numpy as np
import matplotlib.pyplot as plt

func = np.poly1d(np.array([1, 2, 3, 4]).astype(float))
```

```
func1 = func.deriv(m=1)
x = np.linspace(-10, 10, 30)
y = func(x)
y1 = func1(x)

plt.plot(x, y, 'ro', x, y1, 'g--')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

## Subplots

At a certain point, you will have too many lines in one plot. However, you would still like everything grouped together. We can do this with the `subplot()` function. This function creates multiple plots in a grid.

### Time for action – plotting a polynomial and its derivatives

Let's plot a polynomial and its first and second derivative. We will make three subplots for the sake of clarity:

1. Create a polynomial and its derivatives using the following code:

```
func = np.poly1d(np.array([1, 2, 3, 4]).astype(float))
x = np.linspace(-10, 10, 30)
y = func(x)
func1 = func.deriv(m=1)
y1 = func1(x)
func2 = func.deriv(m=2)
y2 = func2(x)
```

2. Create the first subplot of the polynomial with the `subplot()` function. The first parameter of this function is the number of rows, the second parameter is the number of columns, and the third parameter is an index number starting with 1. Alternatively, combine the three parameters into a single number, such as 311. The subplots will be organized in three rows and one column. Give the subplot the title **Polynomial**. Make a solid red line:

```
plt.subplot(311)
plt.plot(x, y, 'r-')
plt.title("Polynomial")
```



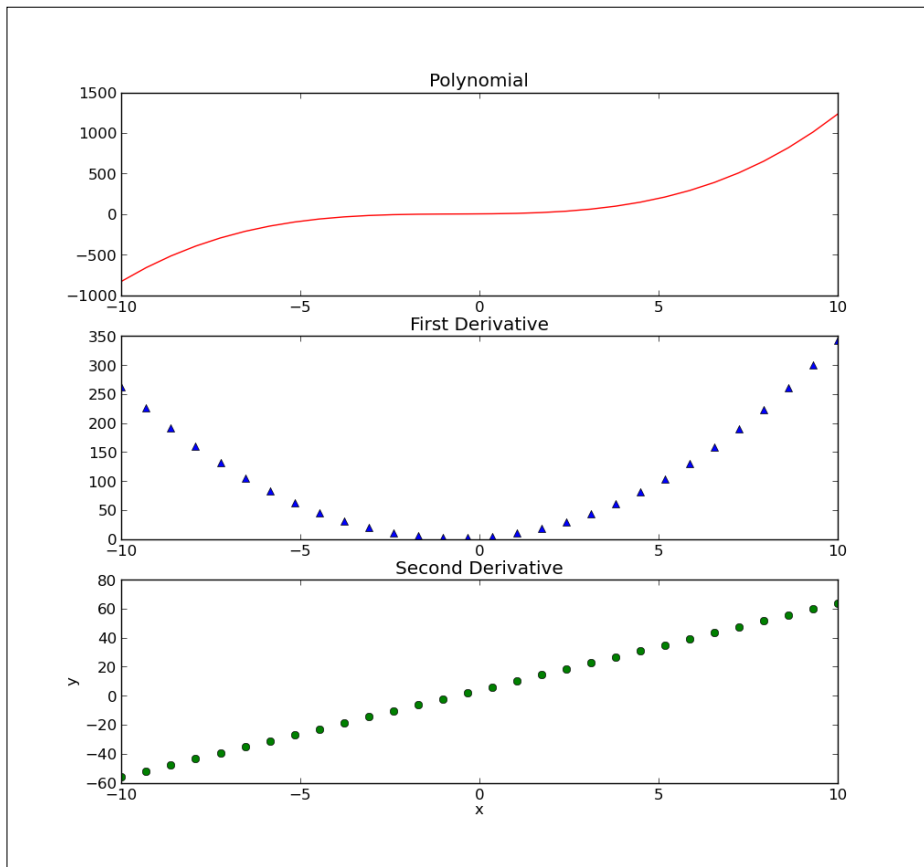
3. Create the third subplot of the first derivative with the `subplot()` function. Give the subplot the title **First Derivative**. Use a line of blue triangles:

```
plt.subplot(312)
plt.plot(x, y1, 'b^')
plt.title("First Derivative")
```

4. Create the second subplot of the second derivative with the `subplot()` function. Give the subplot the title **Second Derivative**. Use a line of green circles:

```
plt.subplot(313)
plt.plot(x, y2, 'go')
plt.title("Second Derivative")
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

The three subplots with polynomial coefficients 1, 2, 3, and 4 are as follows:



## What just happened?

We plotted a polynomial and its first and second derivatives using three different line styles and three subplots in three rows and one column (see `polyplot3.py`):

```
import numpy as np
import matplotlib.pyplot as plt

func = np.poly1d(np.array([1, 2, 3, 4]).astype(float))
x = np.linspace(-10, 10, 30)
y = func(x)
func1 = func.deriv(m=1)
y1 = func1(x)
func2 = func.deriv(m=2)
y2 = func2(x)

plt.subplot(311)
plt.plot(x, y, 'r-')
plt.title("Polynomial")
plt.subplot(312)
plt.plot(x, y1, 'b^')
plt.title("First Derivative")
plt.subplot(313)
plt.plot(x, y2, 'go')
plt.title("Second Derivative")
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

## Finance

`matplotlib` can help monitor our stock investments. The `matplotlib.finance` package has utilities with which we can download stock quotes from Yahoo Finance at <http://finance.yahoo.com/>. We can then plot the data as candlesticks.

### Time for action – plotting a year's worth of stock quotes

We can plot a year's worth of stock quotes data with the `matplotlib.finance` package. This requires a connection to Yahoo Finance, which is the data source.

1. Determine the start date by subtracting one year from today:

```
from matplotlib.dates import DateFormatter
from matplotlib.dates import DayLocator
from matplotlib.dates import MonthLocator
```

```
from matplotlib.finance import quotes_historical_yahoo
from matplotlib.finance import candlestick
import sys
from datetime import date
import matplotlib.pyplot as plt
today = date.today()
start = (today.year - 1, today.month, today.day)
```

- 2.** We need to create the so-called **locators**. These objects from the `matplotlib.dates` package locate months and days on the `x` axis:

```
alldays = DayLocator()
months = MonthLocator()
```

- 3.** Create a date formatter to format the dates on the `x` axis. This formatter creates a string containing the short name of a month and the year:

```
month_formatter = DateFormatter("%b %Y")
```

- 4.** Download the stock quote data from Yahoo finance with the following code:

```
quotes = quotes_historical_yahoo(symbol, start, today)
```

- 5.** Create a `matplotlib` **Figure** object—this is a top-level container for plot components:

```
fig = plt.figure()
```

- 6.** Add a subplot to the figure:

```
ax = fig.add_subplot(111)
```

- 7.** Set the major locator on the `x` axis to the months locator. This locator is responsible for the big ticks on the `x` axis:

```
ax.xaxis.set_major_locator(months)
```

- 8.** Set the minor locator on the `x` axis to the days locator. This locator is responsible for the small ticks on the `x` axis:

```
ax.xaxis.set_minor_locator(alldays)
```

- 9.** Set the major formatter on the `x` axis to the months formatter. This formatter is responsible for the labels of the big ticks on the `x` axis:

```
ax.xaxis.set_major_formatter(month_formatter)
```

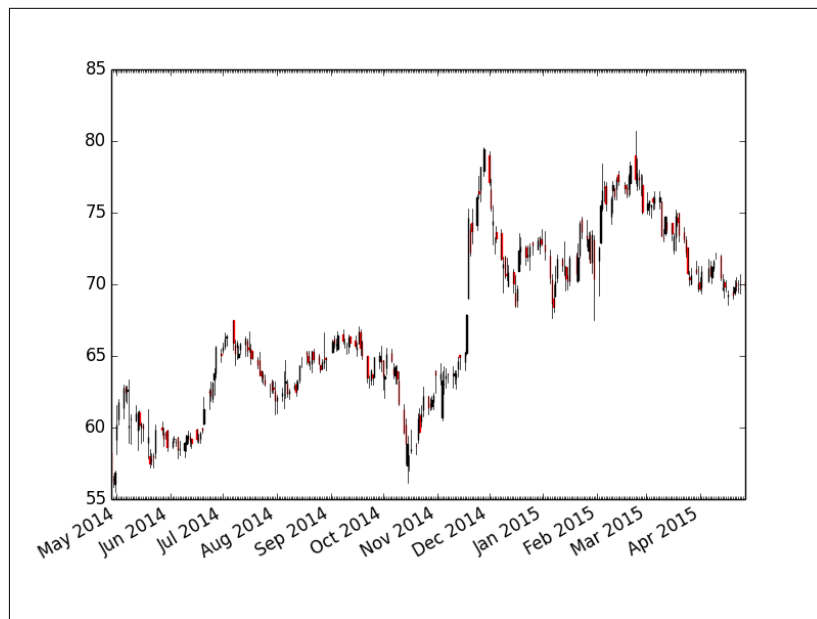
- 10.** A function in the `matplotlib.finance` package allows us to display candlesticks. Create the candlesticks using the quotes data. It is possible to specify the width of the candlesticks. For now, use the default value:

```
candlestick(ax, quotes)
```

- 11.** Format the labels on the x axis as dates. This rotates the labels on the x axis so that they fit better:

```
fig.autofmt_xdate()
plt.show()
```

The candlestick chart for **DISH (Dish Network Corp)** appears as follows:



### ***What just happened?***

We downloaded a year's worth of data from Yahoo Finance. We charted this data using candlesticks (see `candlesticks.py`):

```
from matplotlib.dates import DateFormatter
from matplotlib.dates import DayLocator
from matplotlib.dates import MonthLocator
from matplotlib.finance import quotes_historical_yahoo
from matplotlib.finance import candlestick
import sys
from datetime import date
import matplotlib.pyplot as plt

today = date.today()
start = (today.year - 1, today.month, today.day)
```

```
alldays = DayLocator()
months = MonthLocator()
month_formatter = DateFormatter("%b %Y")

symbol = 'DISH'

if len(sys.argv) == 2:
    symbol = sys.argv[1]

quotes = quotes_historical_yahoo(symbol, start, today)

fig = plt.figure()
ax = fig.add_subplot(111)
ax.xaxis.set_major_locator(months)
ax.xaxis.set_minor_locator(alldays)
ax.xaxis.set_major_formatter(month_formatter)

candlestick(ax, quotes)
fig.autofmt_xdate()
plt.show()
```

## Histograms

Histograms visualize the distribution of numerical data. `matplotlib` has the handy `hist()` function that graphs histograms. The `hist()` function has two main arguments—the array containing the data and the number of bars.

### Time for action – charting stock price distributions

Let's chart the stock price distribution of quotes from Yahoo Finance.

1. Download the data going back one year:

```
today = date.today()
start = (today.year - 1, today.month, today.day)

quotes = quotes_historical_yahoo(symbol, start, today)
```

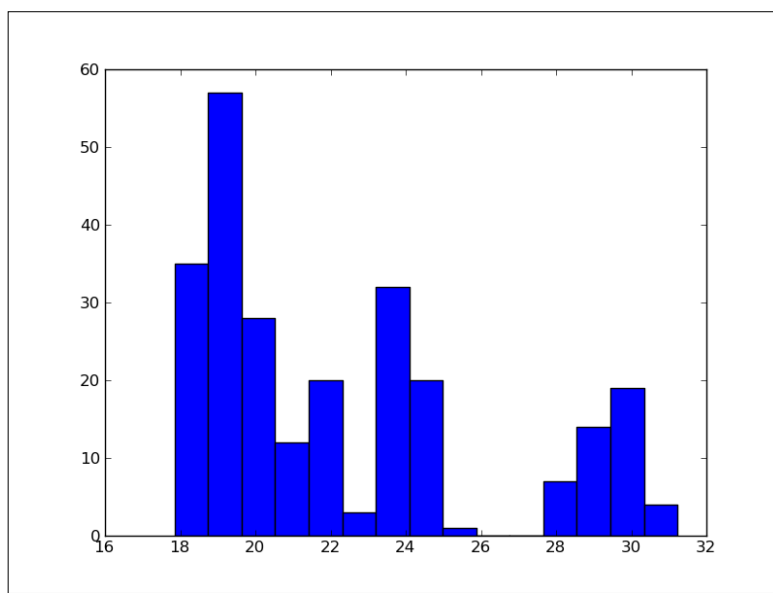
2. The quotes data in the previous step is stored in a Python list. Convert this to a NumPy array and extract the close prices:

```
quotes = np.array(quotes)
close = quotes.T[4]
```

**3.** Draw the histogram with a reasonable number of bars:

```
plt.hist(close, np.sqrt(len(close)))  
plt.show()
```

The histogram for DISH appears as follows:

***What just happened?***

We charted the stock price distribution of DISH as a histogram (see `stockhistogram.py`):

```
from matplotlib.finance import quotes_historical_yahoo  
import sys  
from datetime import date  
import matplotlib.pyplot as plt  
import numpy as np  
  
today = date.today()  
start = (today.year - 1, today.month, today.day)  
  
symbol = 'DISH'  
  
if len(sys.argv) == 2:  
    symbol = sys.argv[1]
```

```
quotes = quotes_historical_yahoo(symbol, start, today)
quotes = np.array(quotes)
close = quotes.T[4]

plt.hist(close, np.sqrt(len(close)))
plt.show()
```

## Have a go hero – drawing a bell curve

Overlay a bell curve (related to the **Gaussian** or normal distribution) using the average price and standard deviation. This is, of course, only an exercise.

## Logarithmic plots

Logarithmic plots are useful when the data has a wide range of values. `matplotlib` has the functions `semilogx()` (logarithmic x axis), `semilogy()` (logarithmic y axis), and `loglog()` (x and y axes logarithmic).

## Time for action – plotting stock volume

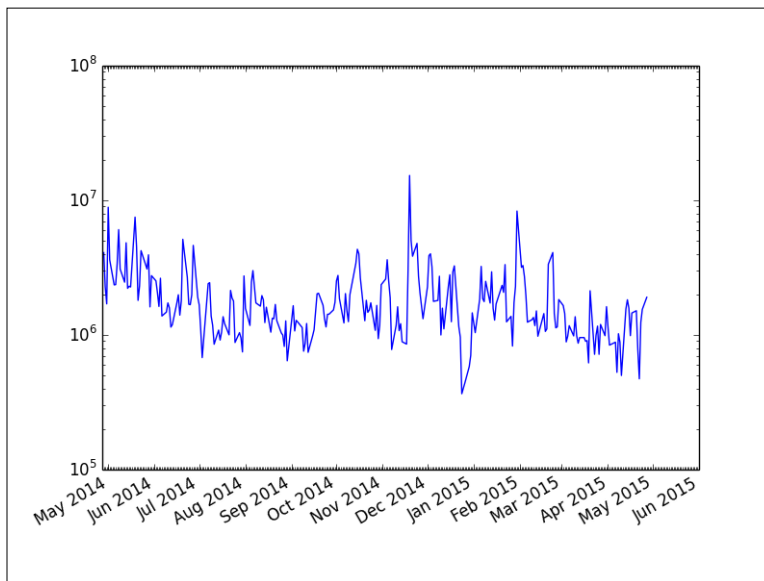
Stock volume varies a lot, so let's plot it on a logarithmic scale. First, we need to download historical data from Yahoo Finance, extract the dates and volume, create locators and a date formatter, and create the figure and add it to a subplot. We already went through these steps in the previous *Time for action* section, so we will skip them here.

Plot the volume using a logarithmic scale:

```
plt.semilogy(dates, volume)
```

Now, set the locators and format the x axis as dates. Instructions for these steps can be found in the previous *Time for action* section as well.

The stock volume using a logarithmic scale for DISH appears as follows:



### ***What just happened?***

We plotted stock volume using a logarithmic scale (see `logy.py`):

```
from matplotlib.finance import quotes_historical_yahoo
from matplotlib.dates import DateFormatter
from matplotlib.dates import DayLocator
from matplotlib.dates import MonthLocator
import sys
from datetime import date
import matplotlib.pyplot as plt
import numpy as np

today = date.today()
start = (today.year - 1, today.month, today.day)

symbol = 'DISH'

if len(sys.argv) == 2:
    symbol = sys.argv[1]
```



```
quotes = quotes_historical_yahoo(symbol, start, today)
quotes = np.array(quotes)
dates = quotes.T[0]
volume = quotes.T[5]

alldays = DayLocator()
months = MonthLocator()
month_formatter = DateFormatter("%b %Y")

fig = plt.figure()
ax = fig.add_subplot(111)
plt.semilogy(dates, volume)
ax.xaxis.set_major_locator(months)
ax.xaxis.set_minor_locator(alldays)
ax.xaxis.set_major_formatter(month_formatter)
fig.autofmt_xdate()
plt.show()
```

## Scatter plots

A scatter plot displays values for two numerical variables in the same dataset. The matplotlib `scatter()` function creates a scatter plot. Optionally, we can specify the color and size of the data points, as well as alpha transparency, in the plot.

### Time for action – plotting price and volume returns with a scatter plot

We can easily make a scatter plot of the stock price and volume returns. Again, let's download the necessary data from Yahoo Finance.

1. The quotes data in the previous step is stored in a Python list. Convert this to a NumPy array and extract the close and volume values:

```
dates = quotes.T[4]
volume = quotes.T[5]
```

2. Calculate the close price and volume returns:

```
ret = np.diff(close)/close[:-1]
volchange = np.diff(volume)/volume[:-1]
```

3. Create a matplotlib figure object:

```
fig = plt.figure()
```

**4.** Add a subplot to the figure:

```
ax = fig.add_subplot(111)
```

**5.** Create the scatter plot with the color of the data points linked to the close return, and the size linked to the volume change:

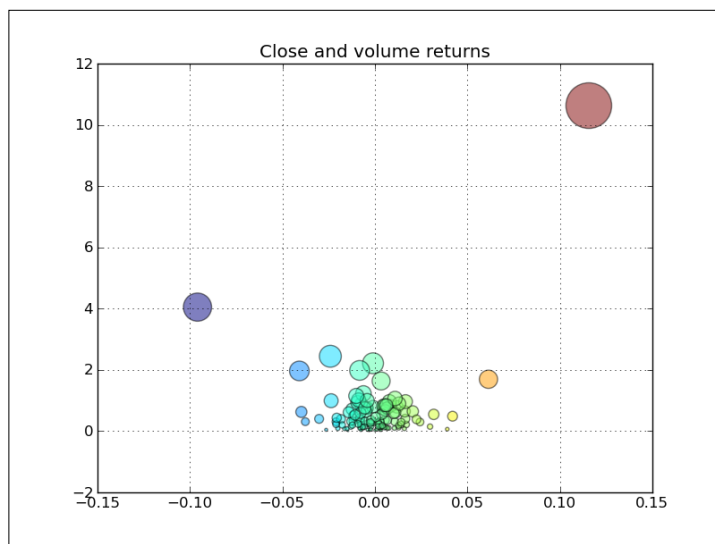
```
ax.scatter(ret, volchange, c=ret * 100,
           s=volchange * 100, alpha=0.5)
```

**6.** Set the title of the plot and put a grid on it:

```
ax.set_title('Close and volume returns')
ax.grid(True)
```

```
plt.show()
```

The scatter plot for DISH appears as follows:

***What just happened?***

We made a scatter plot of the close price and volume returns for DISH (see `scatterprice.py`):

```
from matplotlib.finance import quotes_historical_yahoo
import sys
from datetime import date
import matplotlib.pyplot as plt
import numpy as np
```

```
today = date.today()
start = (today.year - 1, today.month, today.day)

symbol = 'DISH'

if len(sys.argv) == 2:
    symbol = sys.argv[1]

quotes = quotes_historical_yahoo(symbol, start, today)
quotes = np.array(quotes)
close = quotes.T[4]
volume = quotes.T[5]
ret = np.diff(close)/close[:-1]
volchange = np.diff(volume)/volume[:-1]

fig = plt.figure()
ax = fig.add_subplot(111)
ax.scatter(ret, volchange, c=ret * 100, s=volchange * 100, alpha=0.5)
ax.set_title('Close and volume returns')
ax.grid(True)

plt.show()
```

## Fill between

The `fill_between()` function fills a plot region with a specified color. We can choose an optional alpha channel value. The function also has a `where` parameter so that we can shade a region based on a condition.

### Time for action – shading plot regions based on a condition

Imagine that you want to shade a region of a stock chart, where the closing price is below average, with a different color than when it is above the mean. The `fill_between()` function is the best choice for the job. We will, again, omit the steps of downloading historical data going back one year, extracting dates and close prices, and creating locators and date formatter.

1. Create a matplotlib Figure object:

```
fig = plt.figure()
```

2. Add a subplot to the figure:

```
ax = fig.add_subplot(111)
```

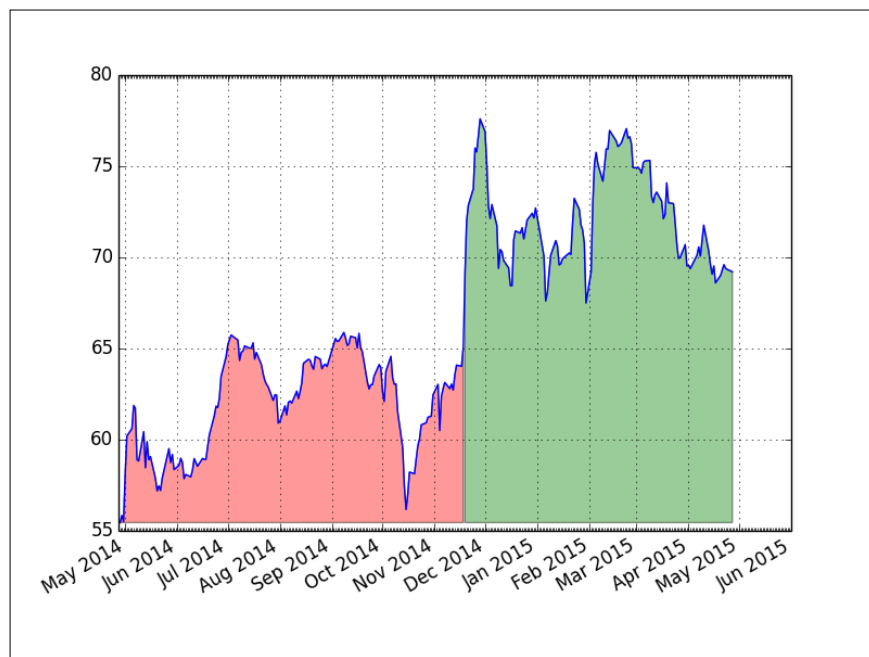
**3.** Plot the closing price:

```
ax.plot(dates, close)
```

**4.** Shade the regions of the plot below the closing price using different colors depending on whether the values are below or above the average price:

```
plt.fill_between(dates, close.min(), close,
                 where=close>close.mean(), facecolor="green", alpha=0.4)
plt.fill_between(dates, close.min(), close,
                 where=close<close.mean(), facecolor="red", alpha=0.4)
```

Now we can finish the plot as shown by setting locators and formatting the x axis values as dates. The stock price using conditional shading for DISH is as follows:

***What just happened?***

We shaded the region of a stock chart, where the closing price is below average, with a different color than when it is above the mean (see `fillbetween.py`):

```
from matplotlib.finance import quotes_historical_yahoo
from matplotlib.dates import DateFormatter
from matplotlib.dates import DayLocator
from matplotlib.dates import MonthLocator
import sys
```

```
from datetime import date
import matplotlib.pyplot as plt
import numpy as np

today = date.today()
start = (today.year - 1, today.month, today.day)

symbol = 'DISH'

if len(sys.argv) == 2:
    symbol = sys.argv[1]

quotes = quotes_historical_yahoo(symbol, start, today)
quotes = np.array(quotes)
dates = quotes.T[0]
close = quotes.T[4]

alldays = DayLocator()
months = MonthLocator()
month_formatter = DateFormatter("%b %Y")

fig = plt.figure()
ax = fig.add_subplot(111)
ax.plot(dates, close)
plt.fill_between(dates, close.min(), close, where=close>close.mean(),
                facecolor="green", alpha=0.4)
plt.fill_between(dates, close.min(), close, where=close<close.mean(),
                facecolor="red", alpha=0.4)
ax.xaxis.set_major_locator(months)
ax.xaxis.set_minor_locator(alldays)
ax.xaxis.set_major_formatter(month_formatter)
ax.grid(True)
fig.autofmt_xdate()
plt.show()
```

## **Legend and annotations**

Legends and annotations are essential for good plots. We can create transparent legends with the `legend()` function and let `matplotlib` figure out where to place them. Also, with the `annotate()` function, we can accurately annotate on a plot. There are a large number of annotation and arrow styles.

## Time for action – using a legend and annotations

In *Chapter 3, Getting Familiar with Commonly Used Functions*, we learned how to calculate the EMA of stock prices. We will plot the close price of a stock and three of its EMA. To clarify the plot, we will add a legend. We will also indicate crossovers of two of the averages with annotations. Some steps are again omitted to avoid repetition.

1. Go back to *Chapter 3, Getting Familiar with Commonly Used Functions*, if needed, and review the EMA algorithm. Calculate and plot the EMAs of 9, 12, and 15 periods:

```
emas = []

for i in range(9, 18, 3):
    weights = np.exp(np.linspace(-1., 0., i))
    weights /= weights.sum()

    ema = np.convolve(weights, close)[i-1:-i+1]
    idx = (i - 6)/3
    ax.plot(dates[i-1:], ema, lw=idx, label="EMA(%s)" % (i))
    data = np.column_stack((dates[i-1:], ema))
    emas.append(np.rec.fromrecords(
        data, names=["dates", "ema"]))
```

Notice that the `plot()` function call needs a label for the legend. We stored the moving averages in record arrays for the next step.

2. Let's find the crossover points of the first two moving averages:

```
first = emas[0]["ema"].flatten()
second = emas[1]["ema"].flatten()
bools = np.abs(first[-len(second):] - second)/second < 0.0001
xpoints = np.compress(bools, emas[1])
```

3. Now that we have the crossover points, annotate them with arrows. Make sure that the annotation text is slightly away from the crossover points:

```
for xpoint in xpoints:
    ax.annotate('x', xy=xpoint, textcoords='offset points',
               xytext=(-50, 30),
               arrowprops=dict(arrowstyle="->"))
```

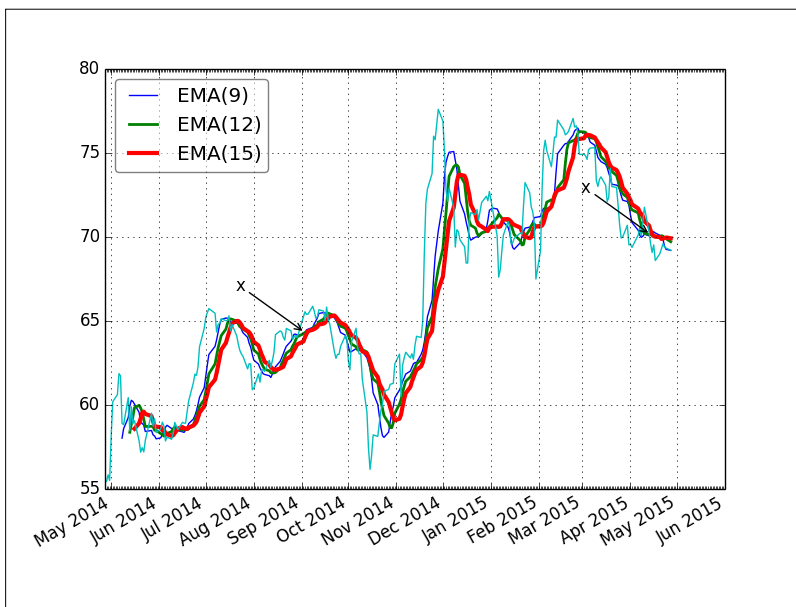
4. Add a legend and let `matplotlib` decide where to put it:

```
leg = ax.legend(loc='best', fancybox=True)
```

**5.** Make the legend transparent by setting the alpha channel value:

```
leg.get_frame().set_alpha(0.5)
```

The stock price and moving averages with a legend and annotations appears as follows:



### ***What just happened?***

We plotted the close price of a stock and three of its EMAs. We added a legend to the plot. We annotated the crossover points of the first two averages with annotations (see `emallegend.py`):

```
from matplotlib.finance import quotes_historical_yahoo
from matplotlib.dates import DateFormatter
from matplotlib.dates import DayLocator
from matplotlib.dates import MonthLocator
import sys
from datetime import date
import matplotlib.pyplot as plt
import numpy as np

today = date.today()
start = (today.year - 1, today.month, today.day)
```

```
symbol = 'DISH'

if len(sys.argv) == 2:
    symbol = sys.argv[1]

quotes = quotes_historical_yahoo(symbol, start, today)
quotes = np.array(quotes)
dates = quotes.T[0]
close = quotes.T[4]

fig = plt.figure()
ax = fig.add_subplot(111)

emas = []
for i in range(9, 18, 3):
    weights = np.exp(np.linspace(-1., 0., i))
    weights /= weights.sum()
    ema = np.convolve(weights, close)[i-1:-i+1]
    idx = (i - 6)/3
    ax.plot(dates[i-1:], ema, lw=idx, label="EMA(%s)" % (i))
    data = np.column_stack((dates[i-1:], ema))
    emas.append(np.rec.fromrecords(data, names=["dates", "ema"]))

first = emas[0]["ema"].flatten()
second = emas[1]["ema"].flatten()
bools = np.abs(first[-len(second):] - second)/second < 0.0001
xpoints = np.compress(bools, emas[1])

for xpoint in xpoints:
    ax.annotate('x', xy=xpoint, textcoords='offset points',
               xytext=(-50, 30),
               arrowprops=dict(arrowstyle="->"))

leg = ax.legend(loc='best', fancybox=True)
leg.get_frame().set_alpha(0.5)

alldays = DayLocator()
months = MonthLocator()
month_formatter = DateFormatter("%b %Y")
ax.plot(dates, close, lw=1.0, label="Close")
ax.xaxis.set_major_locator(months)
ax.xaxis.set_minor_locator(alldays)
```



```
ax.xaxis.set_major_formatter(month_formatter)
ax.grid(True)
fig.autofmt_xdate()
plt.show()
```

## Three-dimensional plots

Three-dimensional plots are pretty spectacular, so we have to cover them here too. For three-dimensional plots, we need an `Axes3D` object associated with a 3D projection.

### Time for action – plotting in three dimensions

We will plot a simple three-dimensional function:

$$z = x^2 + y^2$$

1. Use the 3D keyword to specify a three-dimensional projection for the plot:  

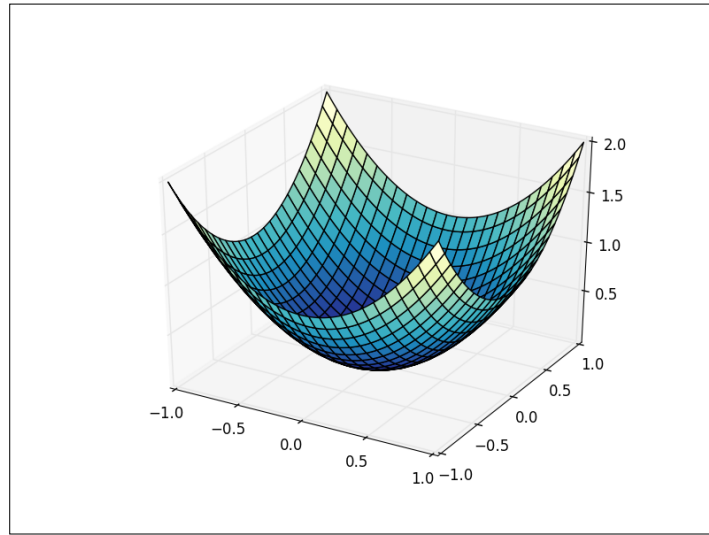
```
ax = fig.add_subplot(111, projection='3d')
```
2. To create a square two-dimensional grid, use the `meshgrid()` function to initialize the `x` and `y` values:  

```
u = np.linspace(-1, 1, 100)

x, y = np.meshgrid(u, u)
```
3. We will specify the row strides, column strides, and the color map for the surface plot. The strides determine the size of the tiles in the surface. The choice for color map is a matter of taste:  

```
ax.plot_surface(x, y, z, rstride=4, cstride=4,
               cmap=cm.YlGnBu_r)
```

The result is the following three-dimensional plot:



### ***What just happened?***

We created a plot of a three-dimensional function (see `three_d.py`):

```
from mpl_toolkits.mplot3d import Axes3D
import matplotlib.pyplot as plt
import numpy as np
from matplotlib import cm

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

u = np.linspace(-1, 1, 100)

x, y = np.meshgrid(u, u)
z = x ** 2 + y ** 2
ax.plot_surface(x, y, z, rstride=4,
               cstride=4, cmap=cm.YlGnBu_r)

plt.show()
```

## Contour plots

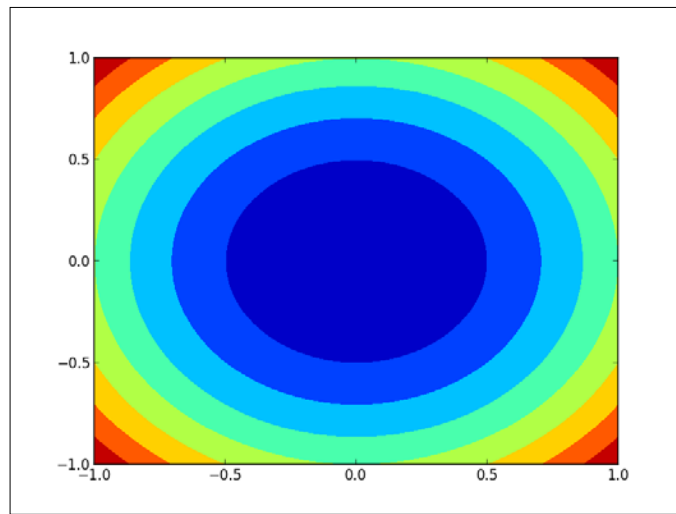
matplotlib contour three-dimensional plots come in two flavors—filled and unfilled. Contour plots use the so-called **contour lines**. You may be familiar with contour lines from geographic maps. In such maps, contour lines connect points of the same elevation above sea level. We can create normal contour plots with the `contour()` function. For filled contour plots, we use the `contourf()` function.

### Time for action – drawing a filled contour plot

We will draw a filled contour plot of the three-dimensional mathematical function in the previous *Time for action* section. The code is also pretty similar. One key difference is that we don't need the 3D projection parameter any more. To draw the filled contour plot, use the following line of code:

```
ax.contourf(x, y, z)
```

This gives us the following filled contour plot:



### What just happened?

We created a filled contour plot of a three-dimensional mathematical function (see `contour.py`):

```
import matplotlib.pyplot as plt
import numpy as np
from matplotlib import cm
```

```
fig = plt.figure()
ax = fig.add_subplot(111)

u = np.linspace(-1, 1, 100)

x, y = np.meshgrid(u, u)
z = x ** 2 + y ** 2
ax.contourf(x, y, z)

plt.show()
```

## Animation

`matplotlib` offers fancy animation capabilities via a special animation module. We need to define a callback function that is used to regularly update the screen. We also need a function to generate data to be plotted.

### Time for action – animating plots

We will plot three random datasets and display them as circles, dots, and triangles. However, we will only update two of those datasets with random values.

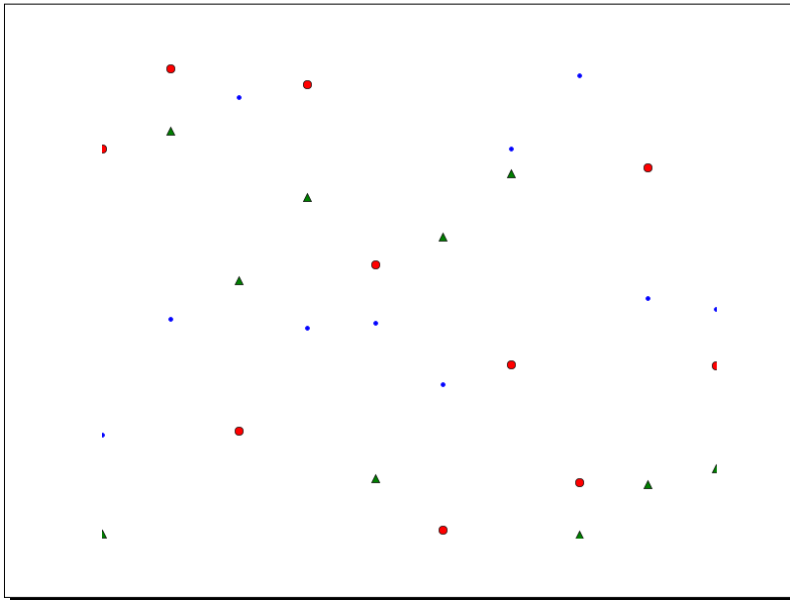
1. Plot three random datasets as circles, dots, and triangles in different colors:  
circles, triangles, dots = ax.plot(x, 'ro', y, 'g^', z, 'b.')
2. This function gets called to update the screen regularly. Update two of the plots with new y values:

```
def update(data):
    circles.set_ydata(data[0])
    triangles.set_ydata(data[1])
    return circles, triangles
```

3. Generate random data with NumPy:

```
def generate():
    while True: yield np.random.rand(2, N)
```

The following is a snapshot of the animation in action:



### ***What just happened?***

We created an animation of random data points (see `animation.py`):

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.animation as animation

fig = plt.figure()
ax = fig.add_subplot(111)
N = 10
x = np.random.rand(N)
y = np.random.rand(N)
z = np.random.rand(N)
circles, triangles, dots = ax.plot(x, 'ro', y, 'g^', z, 'b.')
ax.set_ylim(0, 1)
plt.axis('off')

def update(data):
    circles.set_ydata(data[0])
    triangles.set_ydata(data[1])
    return circles, triangles
```

```
def generate():
    while True: yield np.random.rand(2, N)

anim = animation.FuncAnimation(fig, update,
                               generate, interval=150)
plt.show()
```

## Summary

This chapter was about `matplotlib`—a Python plotting library. We covered simple plots, histograms, plot customization, subplots, three-dimensional plots, contour plots, and logarithmic plots. You also saw a few examples of displaying stock charts. Obviously, we only scratched the surface and just saw the tip of the iceberg. `matplotlib` is very feature rich, so we didn't have space to cover Latex support, polar coordinates support, and other functionality.

The author of `matplotlib`, **John Hunter**, passed away in August 2012. One of the technical reviewers of this book suggested mentioning the **John Hunter Memorial Fund** (<http://numfocus.org/news/2012/08/28/johnhunter/>). The memorial fund set up by the **NumFocus Foundation** is an opportunity for us, fans of John Hunter's work, to "give back". Again, for more details, check out the preceding link to the NumFocus website.

The next chapter is about SciPy—a scientific Python framework that is built on top of NumPy.



# 10

## When NumPy Is Not Enough – SciPy and Beyond

*SciPy is a world famous Python open source scientific computing library built on top of NumPy. It adds functionalities such as numerical integration, optimization, statistics, and special functions.*

In this chapter, we will cover the following topics:

- ◆ File I/O
- ◆ Statistics
- ◆ Signal processing
- ◆ Optimization
- ◆ Interpolation
- ◆ Image and audio processing

### **MATLAB and Octave**

**MATLAB** and its open source alternative, **Octave**, are popular mathematical programs. The `scipy.io` package has functions that let you load **MATLAB** or **Octave matrices** and arrays of numbers or strings in Python programs, and vice versa. The `loadmat()` function loads a `.mat` file. The `savemat()` function saves a dictionary of names and arrays into a `.mat` file.



## Time for action – saving and loading a .mat file

If we start with NumPy arrays and decide to use said arrays within a MATLAB or Octave environment, the easiest thing to do is create a .mat file. We can, then, load the file within MATLAB or Octave. Let's go through the necessary steps:

1. Create a NumPy array and call the `savemat()` function to create a .mat file. This function has two parameters: a file name and a dictionary containing variable names and values:

```
a = np.arange(7)

io.savemat("a.mat", {"array": a})
```

2. Within a MATLAB or Octave environment, load the .mat file and check the stored array:

```
octave-3.4.0:7> load a.mat
octave-3.4.0:8> a
```

```
octave-3.4.0:8> array
array =
```

```
0
1
2
3
4
5
6
```

### What just happened?

We created a .mat file from NumPy code and loaded it within Octave. We checked the NumPy array that was created (see `scipyio.py`):

```
import numpy as np
from scipy import io

a = np.arange(7)

io.savemat("a.mat", {"array": a})
```

## Pop quiz – loading .mat files

Q1. Which function loads .mat files?

1. Loadmatlab
2. loadmat
3. loadoct
4. frommat

## Statistics

The SciPy statistics module is called `scipy.stats`. There is one class that implements continuous distributions and one class that implements discrete distributions. Also, in this module, functions that perform a great number of statistical tests can be found.

## Time for action – analyzing random values

We will generate random values that mimic a normal distribution and analyze the generated data with statistical functions from the `scipy.stats` package.

1. Generate random values from a normal distribution using the `scipy.stats` package:
 

```
generated = stats.norm.rvs(size=900)
```

2. Fit the generated values to a normal distribution. This basically gives the mean and standard deviation of the dataset:

```
print("Mean", "Std", stats.norm.fit(generated))
```

The mean and standard deviation appear as follows:

```
Mean Std (0.0071293257063200707, 0.95537708218972528)
```

3. **Skewness** tells us how skewed (asymmetric) a probability distribution is (see <http://en.wikipedia.org/wiki/Skewness>). Perform a skewness test. This test returns two values. The second value is the **p-value**—the probability that the skewness of the dataset does not correspond to a normal distribution.



Generally speaking, the p-value is the probability of an outcome different than what was expected given the null hypothesis—in this case, the probability of getting a skewness different from that of a normal distribution (which is 0 because of symmetry).

P-values range from 0 to 1:

```
print("Skewtest", "pvalue", stats.skewtest(generated))
```

The result of the skewness test appears as follows:

```
Skewtest pvalue (-0.62120640688766893, 0.5344638245033837)
```

So, there is a 53 percent chance we are not dealing with a normal distribution. It is instructive to see what happens if we generate more points, because if we generate more points, we should have a more normal distribution. For 900,000 points, we get a p-value of 0.16. For 20 generated values, the p-value is 0.50.

- 4. Kurtosis** tells us how curved a probability distribution is. Perform a kurtosis test. This test is set up similarly to the skewness test, but, of course, applies to kurtosis:

```
print("Kurtosistest", "pvalue",  
      stats.kurtosistest(generated))
```

The result of the kurtosis test appears as follows:

```
Kurtosistest pvalue (1.3065381019536981, 0.19136963054975586)
```

The p-value for 900,000 values is 0.028. For 20 generated values, the p-values is 0.88.

- 5. A normality test** tells us how likely it is that a dataset complies the normal distribution. Perform a normality test. This test also returns two values, of which the second is a p-value:

```
print("Normaltest", "pvalue", stats.normaltest(generated))
```

The result of the normality test appears as follows:

```
Normaltest pvalue (2.09293921181506, 0.35117535059841687)
```

The p-value for 900,000 generated values is 0.035. For 20 generated values, the p-value is 0.79.

- 6.** We can find the value at a certain percentile easily with SciPy:

```
print("95 percentile",  
      stats.scoreatpercentile(generated, 95))
```

The value at the 95th percentile appears as follows:

```
95 percentile 1.54048860252
```

- 7.** Do the opposite of the previous step to find the percentile at 1:

```
print("Percentile at 1",  
      stats.percentileofscore(generated, 1))
```

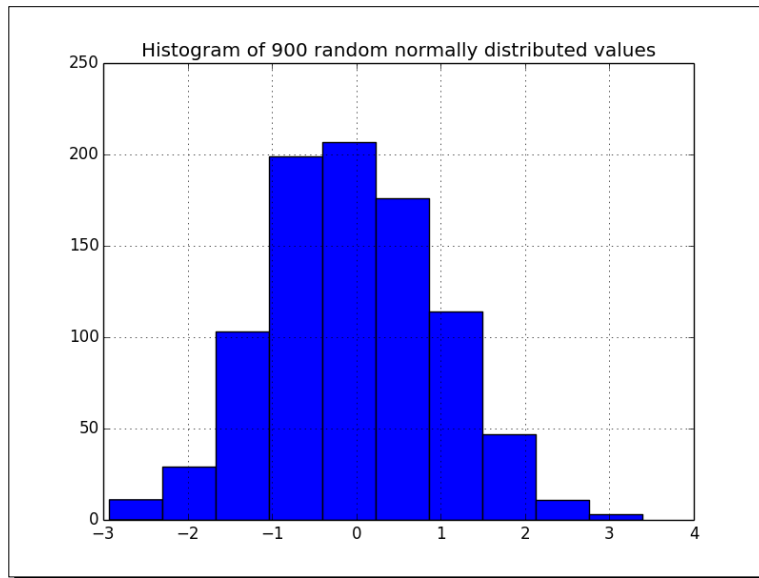
The percentile at 1 appears as follows:

```
Percentile at 1 85.5555555556
```

8. Plot the generated values in a histogram with `matplotlib` (more information about `matplotlib` can be found in the previous *Chapter 9, Plotting with matplotlib*):

```
plt.hist(generated)
```

The histogram of the generated random values is as follows:



### ***What just happened?***

We created a dataset from a normal distribution and analyzed it with the `scipy.stats` module (see `statistics.py`):

```
from __future__ import print_function
from scipy import stats
import matplotlib.pyplot as plt

generated = stats.norm.rvs(size=900)
print("Mean", "Std", stats.norm.fit(generated))
print("Skewtest", "pvalue", stats.skewtest(generated))
print("Kurtosistest", "pvalue", stats.kurtosistest(generated))
print("Normaltest", "pvalue", stats.normaltest(generated))
print("95 percentile", stats.scoreatpercentile(generated, 95))
print("Percentile at 1", stats.percentileofscore(generated, 1))
plt.title('Histogram of 900 random normally distributed values')
plt.hist(generated)
plt.grid()
plt.show()
```

## Have a go hero – improving the data generation

Judging from the histogram in the previous *Time for action* section, there is room for improvement when it comes to generating the data. Try using NumPy or different parameters of the `scipy.stats.norm.rvs()` function.

## Sample comparison and SciKits

Often we have two data samples, maybe from different experiments, that are somehow related. Statistical tests exist that can compare the samples. Some of these are implemented in the `scipy.stats` module.

Another statistical test that I like is the **Jarque–Bera** normality test from `scikits.statsmodels.stattools`. **SciKits** are small experimental Python software toolkits. They are not part of SciPy. There is also pandas, which is an offshoot of `scikits.statsmodels`. A list of SciKits can be found at <https://scikits.appspot.com/scikits>. You can install `statsmodels` using `setuptools` with:

```
$ [sudo] easy_install statsmodels
```

## Time for action – comparing stock log returns

We will download the stock quotes for the last year of two trackers using `matplotlib`. As mentioned in the previous *Chapter 9, Plotting with matplotlib*, we can retrieve quotes from Yahoo Finance. We will compare the log returns of the close price of **DIA** and **SPY** (DIA tracks the Dow Jones index; SPY tracks the S & P 500 index). We will also perform the Jarque–Bera test on the difference of the log returns.

1. Write a function that can return the close price for a specified stock:

```
def get_close(symbol):
    today = date.today()
    start = (today.year - 1, today.month, today.day)

    quotes = quotes_historical_yahoo(symbol, start, today)
    quotes = np.array(quotes)

    return quotes.T[4]
```

2. Calculate the log returns for DIA and SPY. Compute the log returns by taking the natural logarithm of the close price and then taking the difference of consecutive values:

```
spy = np.diff(np.log(get_close("SPY")))
dia = np.diff(np.log(get_close("DIA")))
```


3. The means comparison test checks whether two different samples could have the same mean value. Two values are returned, of which the second is the p-value from 0 to 1:

```
print("Means comparison", stats.ttest_ind(spy, dia))
```

The result of the means comparison test appears as follows:

```
Means comparison (-0.017995865641886155, 0.98564930169871368)
```

So, there is about a 98 percent chance that the two samples have the same mean log return. Actually, the documentation has the following to say:

 If we observe a large p-value, for example, larger than 0.05 or 0.1, then we cannot reject the null hypothesis of identical average scores. If the p-value is smaller than the threshold, e.g. 1%, 5% or 10%, then we reject the null hypothesis of equal averages.

4. The **Kolmogorov–Smirnov** two samples test tells us how likely it is that two samples are drawn from the same distribution:

```
print("Kolmogorov smirnov test", stats.ks_2samp(spy, dia))
```

Again, two values are returned, of which the second value is the p-value:

```
Kolmogorov smirnov test (0.063492063492063516,
0.67615647616238039)
```

5. Unleash the **Jarque–Bera** normality test on the difference of the log returns:

```
print("Jarque Bera test",
      jarque_bera(spy - dia)[1])
```

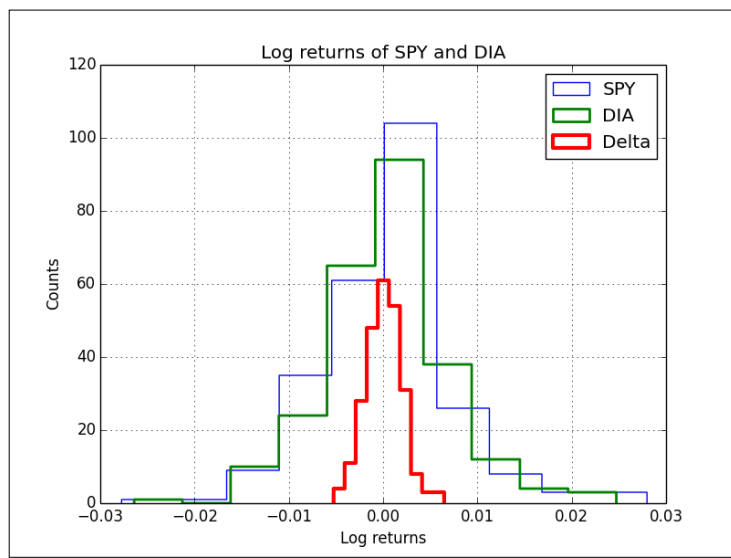
The p-value of the Jarque–Bera normality test appears as follows:

```
Jarque Bera test 0.596125711042
```

6. Plot the histograms of the log returns and the difference thereof with `matplotlib`:

```
plt.hist(spy, histtype="step", lw=1, label="SPY")
plt.hist(dia, histtype="step", lw=2, label="DIA")
plt.hist(spy - dia, histtype="step", lw=3,
         label="Delta")
plt.legend()
plt.show()
```

The histograms of the log returns and difference are shown as follows:



### ***What just happened?***

We compared samples of log returns for DIA and SPY. Also, we performed the Jarque-Bera test on the difference of the log returns (see `pair.py`):

```
from __future__ import print_function
from matplotlib.finance import quotes_historical_yahoo
from datetime import date
import numpy as np
from scipy import stats
from statsmodels.stats.stattools import jarque_bera
import matplotlib.pyplot as plt

def get_close(symbol):
    today = date.today()
    start = (today.year - 1, today.month, today.day)
    quotes = quotes_historical_yahoo(symbol, start, today)
    quotes = np.array(quotes)
    return quotes.T[4]

spy = np.diff(np.log(get_close("SPY")))
dia = np.diff(np.log(get_close("DIA")))
```

```

print("Means comparison", stats.ttest_ind(spy, dia))
print("Kolmogorov smirnov test", stats.ks_2samp(spy, dia))

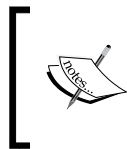
print("Jarque Bera test", jarque_bera(spy - dia)[1])

plt.title('Log returns of SPY and DIA')
plt.hist(spy, histtype="step", lw=1, label="SPY")
plt.hist(dia, histtype="step", lw=2, label="DIA")
plt.hist(spy - dia, histtype="step", lw=3, label="Delta")
plt.xlabel('Log returns')
plt.ylabel('Counts')
plt.grid()
plt.legend(loc='best')
plt.show()

```

## Signal processing

The `scipy.signal` module contains filter functions and **B-spline interpolation** algorithms.



Spline interpolation uses a polynomial called a spline for interpolation (see [http://en.wikipedia.org/wiki/Spline\\_interpolation](http://en.wikipedia.org/wiki/Spline_interpolation)). The interpolation then tries to glue splines together to fit the data. B-spline is a type of spline.

A SciPy signal is defined as an array of numbers. An example of a filter is the `detrend()` function. This function takes a signal and does a linear fit on it. This trend is then subtracted from the original input data.

### Time for action – detecting a trend in QQQ

Often we are more interested in the trend of a data sample than in detrending it. We can still get the trend back easily after detrending. Let's do that for one year of price data for QQQ.

1. Write code that gets the close price and corresponding dates for QQQ:

```

today = date.today()
start = (today.year - 1, today.month, today.day)

quotes = quotes_historical_yahoo("QQQ", start, today)
quotes = np.array(quotes)

dates = quotes.T[0]
qqq = quotes.T[4]

```



**2.** Detrend the signal:

```
y = signal.detrend(qqq)
```

**3.** Create month and day locators for the dates:

```
alldays = DayLocator()  
months = MonthLocator()
```

**4.** Create a date formatter that creates a string of month name and year:

```
month_formatter = DateFormatter("%b %Y")
```

**5.** Create a figure and subplot:

```
fig = plt.figure()  
ax = fig.add_subplot(111)
```

**6.** Plot the data and underlying trend by subtracting the detrended signal:

```
plt.plot(dates, qqq, 'o', dates, qqq - y, '-')
```

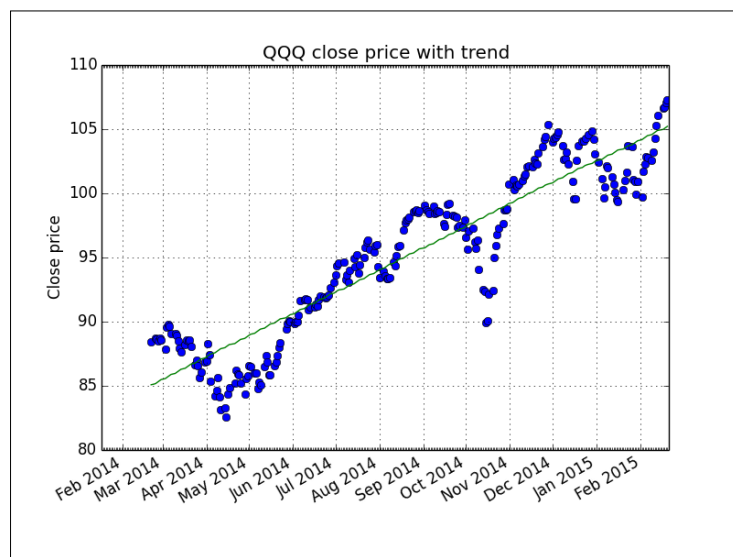
**7.** Set the locators and formatter:

```
ax.xaxis.set_minor_locator(alldays)  
ax.xaxis.set_major_locator(months)  
ax.xaxis.set_major_formatter(month_formatter)
```

**8.** Format the x-axis labels as dates:

```
fig.autofmt_xdate()  
plt.show()
```

The following figure shows the QQQ prices with a trend line:



## ***What just happened?***

We plotted the closing price for QQQ with a trend line (see `trend.py`):

```
from matplotlib.finance import quotes_historical_yahoo
from datetime import date
import numpy as np
from scipy import signal
import matplotlib.pyplot as plt
from matplotlib.dates import DateFormatter
from matplotlib.dates import DayLocator
from matplotlib.dates import MonthLocator

today = date.today()
start = (today.year - 1, today.month, today.day)

quotes = quotes_historical_yahoo("QQQ", start, today)
quotes = np.array(quotes)

dates = quotes.T[0]
qqq = quotes.T[4]

y = signal.detrend(qqq)

alldays = DayLocator()
months = MonthLocator()
month_formatter = DateFormatter("%b %Y")

fig = plt.figure()
ax = fig.add_subplot(111)

plt.title('QQQ close price with trend')
plt.ylabel('Close price')
plt.plot(dates, qqq, 'o', dates, qqq - y, '-')
ax.xaxis.set_minor_locator(alldays)
ax.xaxis.set_major_locator(months)
ax.xaxis.set_major_formatter(month_formatter)
fig.autofmt_xdate()
plt.grid()
plt.show()
```

## Fourier analysis

Signals in the real world often have a periodic nature. A commonly used tool to deal with these signals is the **Discrete Fourier transform** (see [https://en.wikipedia.org/wiki/Discrete-time\\_Fourier\\_transform](https://en.wikipedia.org/wiki/Discrete-time_Fourier_transform)). The Discrete Fourier transform is a transformation from the time domain into the frequency domain, that is, the linear decomposition of a periodic signal into sine and cosine functions with various frequencies:

$$\sum_{t=-\infty}^{\infty} x[t] e^{-i\omega t}$$

Functions for Fourier transforms can be found in the `scipy.fftpack` module (NumPy also has its own Fourier package `numpy.fft`). Included in the package are Fast Fourier transforms, differential and pseudo-differential operators, as well as several helper functions. MATLAB users will be pleased to know that a number of functions in the `scipy.fftpack` module have the same name as their MATLAB counterparts, and a similar function as their MATLAB equivalents.

### Time for action – filtering a detrended signal

We learned in the previous *Time for action* section how to detrend a signal. This detrended signal could have a cyclical component. Let's try to visualize this. Some of the steps are a repetition of steps in the previous *Time for action* section, such as downloading the data and setting up `matplotlib` objects. These steps are omitted here.

1. Apply the Fourier transform, giving us the frequency spectrum:  

```
amps = np.abs(fftpack.fftshift(fftpack.rfft(y)))
```
2. Filter out the noise. Let's say, if the magnitude of a frequency component is below 10 percent of the strongest component, throw it out:  

```
amps[amps < 0.1 * amps.max()] = 0
```
3. Transform the filtered signal back to the original domain and plot it together with the detrended signal:  

```
plt.plot(dates, y, 'o', label="detrended")
plt.plot(dates,
         -fftpack.irfft(fftpack.ifftshift(amps)),
         label="filtered")
```
4. Format the x-axis labels as dates and add a legend with extra large size:  

```
fig.autofmt_xdate()
plt.legend(prop={'size': 'x-large'})
```

5. Add a second subplot and plot the frequency spectrum after filtering:

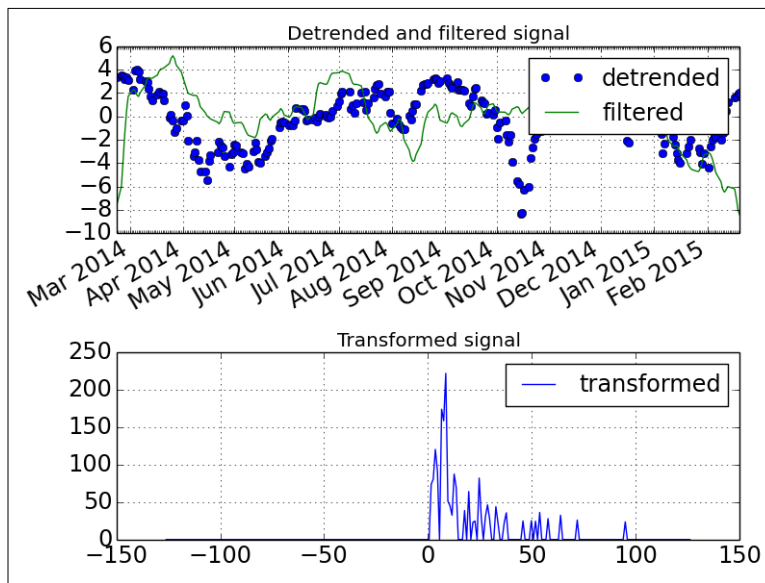
```
ax2 = fig.add_subplot(212)
N = len(qqq)
plt.plot(np.linspace(-N/2, N/2, N), amps,
         label="transformed")
```

6. Display the legend and plot:

```
plt.legend(prop={'size': 'x-large'})
```

```
plt.show()
```

The following plots are of the signal and frequency spectrum:



### ***What just happened?***

We detrended a signal and applied a simple filter on it using the `scipy.fftpack` module (see `frequencies.py`):

```
from matplotlib.finance import quotes_historical_yahoo
from datetime import date
import numpy as np
from scipy import signal
import matplotlib.pyplot as plt
from scipy import fftpack
from matplotlib.dates import DateFormatter
```

```
from matplotlib.dates import DayLocator
from matplotlib.dates import MonthLocator

today = date.today()
start = (today.year - 1, today.month, today.day)

quotes = quotes_historical_yahoo("QQQ", start, today)
quotes = np.array(quotes)

dates = quotes.T[0]
qqq = quotes.T[4]

y = signal.detrend(qqq)

alldays = DayLocator()
months = MonthLocator()
month_formatter = DateFormatter("%b %Y")

fig = plt.figure()
fig.subplots_adjust(hspace=.3)
ax = fig.add_subplot(211)

ax.xaxis.set_minor_locator(alldays)
ax.xaxis.set_major_locator(months)
ax.xaxis.set_major_formatter(month_formatter)

# make font size bigger
ax.tick_params(axis='both', which='major', labelsize='x-large')

amps = np.abs(fftpack.fftshift(fftpack.rfft(y)))
amps[amps < 0.1 * amps.max()] = 0

plt.title('Detrended and filtered signal')
plt.plot(dates, y, 'o', label="detrended")
plt.plot(dates, -fftpack.irfft(fftpack.ifftshift(amps)),
         label="filtered")
fig.autofmt_xdate()
plt.legend(prop={'size': 'x-large'})
plt.grid()
```

```

ax2 = fig.add_subplot(212)
plt.title('Transformed signal')
ax2.tick_params(axis='both', which='major', labelsize='x-large')
N = len(qqq)
plt.plot(np.linspace(-N/2, N/2, N), amps, label="transformed")

plt.legend(prop={'size': 'x-large'})
plt.grid()
plt.tight_layout()
plt.show()

```

## Mathematical optimization

Optimization algorithms try to find the optimal solution for a problem, for instance, finding the maximum or the minimum of a function. The function can be linear or non-linear. The solution could also have special constraints. For example, the solution may not be allowed to have negative values. The `scipy.optimize` module provides several optimization algorithms. One of the algorithms is a least squares fitting function, `leastsq()`. When calling this function, we provide a residuals (error terms) function. This function minimizes the sum of the squares of the residuals; it corresponds to our mathematical model for the solution. It is also necessary to give the algorithm a starting point. This should be a best guess—as close as possible to the real solution. Otherwise, execution will stop after about  $100 * (N+1)$  iterations, where  $N$  is the number of parameters to optimize.

### Time for action – fitting to a sine

In the previous *Time for action* section, we created a simple filter for detrended data. Now, let's use a more restrictive filter that will leave us only with the main frequency component. We will fit a sinusoidal pattern to it and plot our results. This model has four parameters—amplitude, frequency, phase, and vertical offset.

1. Define a residuals function based on a sine wave model:

```

def residuals(p, y, x):
    A,k,theta,b = p
    err = y-A * np.sin(2* np.pi* k * x + theta) + b
    return err

```

2. Transform the filtered signal back to the original domain:

```

filtered = -fftpack.irfft(fftpack.ifftshift(amps))

```

- 3.** Guess the values of the parameters of which we are trying to estimate a transformation from the time domain into the frequency domain:

```
N = len(qqq)
f = np.linspace(-N/2, N/2, N)
p0 = [filtered.max(), f[amps.argmax()]/(2*N), 0, 0]
print("P0", p0)
```

The initial values appear as follows:

```
P0 [2.6679532410065212, 0.00099598469163686377, 0, 0]
```

- 4.** Call the `leastsq()` function:

```
plsq = optimize.leastsq(residuals, p0, args=(filtered,
      dates))
p = plsq[0]
print("P", p)
```

The final parameter values are as follows:

```
P [ 2.67678014e+00  2.73033206e-03 -8.00007036e+03
   -5.01260321e-03]
```

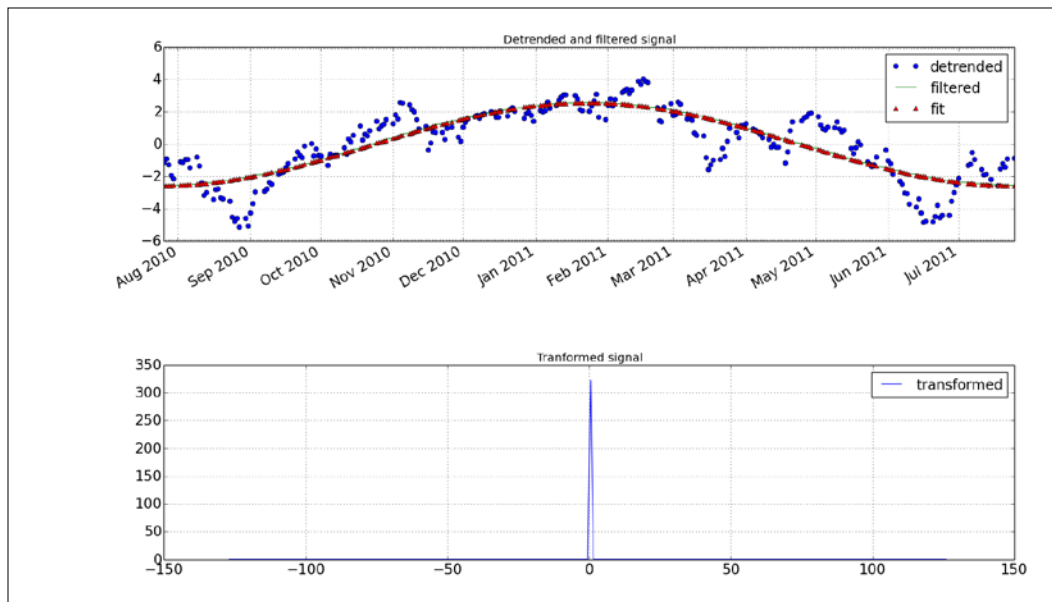
- 5.** Finish the first subplot with detrended data, filtered data, and fit of the filtered data. Use a date format for the horizontal axis and add a legend:

```
plt.plot(dates, y, 'o', label="detrended")
plt.plot(dates, filtered, label="filtered")
plt.plot(dates, p[0] * np.sin(2 * np.pi *
      dates * p[1] + p[2]) + p[3], '^', label="fit")
fig.autofmt_xdate()
plt.legend(prop={'size': 'x-large'})
```

- 6.** Add a second subplot with a legend of the main component of the frequency spectrum:

```
ax2 = fig.add_subplot(212)
plt.plot(f, amps, label="transformed")
```

The following are the resulting charts:



### ***What just happened?***

We detrended one year of price data for QQQ. This signal was then filtered until only the main component of the frequency spectrum was left over. We fitted a sine to the filtered signal using the `scipy.optimize` module (see `optfit.py`):

```
from __future__ import print_function
from matplotlib.finance import quotes_historical_yahoo
import numpy as np
import matplotlib.pyplot as plt
from scipy import fftpack
from scipy import signal
from matplotlib.dates import DateFormatter
from matplotlib.dates import DayLocator
from matplotlib.dates import MonthLocator
from scipy import optimize

start = (2010, 7, 25)
end = (2011, 7, 25)
```



```
quotes = quotes_historical_yahoo("QQQ", start, end)
quotes = np.array(quotes)

dates = quotes.T[0]
qqq = quotes.T[4]

y = signal.detrend(qqq)

alldays = DayLocator()
months = MonthLocator()
month_formatter = DateFormatter("%b %Y")

fig = plt.figure()
fig.subplots_adjust(hspace=.3)
ax = fig.add_subplot(211)

ax.xaxis.set_minor_locator(alldays)
ax.xaxis.set_major_locator(months)
ax.xaxis.set_major_formatter(month_formatter)
ax.tick_params(axis='both', which='major', labelsize='x-large')

amps = np.abs(fftpack.fftshift(fftpack.rfft(y)))
amps[amps < amps.max()] = 0

def residuals(p, y, x):
    A,k,theta,b = p
    err = y-A * np.sin(2* np.pi* k * x + theta) + b
    return err

filtered = -fftpack.irfft(fftpack.ifftshift(amps))
N = len(qqq)
f = np.linspace(-N/2, N/2, N)
p0 = [filtered.max(), f[amps.argmax()]/(2*N), 0, 0]
print("P0", p0)

plsq = optimize.leastsq(residuals, p0, args=(filtered, dates))
p = plsq[0]
print("P", p)
plt.title('Detrended and filtered signal')
plt.plot(dates, y, 'o', label="detrended")
plt.plot(dates, filtered, label="filtered")
```

```

plt.plot(dates, p[0] * np.sin(2 * np.pi * dates * p[1] + p[2]) + p[3],
        '^', label="fit")
fig.autofmt_xdate()
plt.legend(prop={'size': 'x-large'})
plt.grid()

ax2 = fig.add_subplot(212)
plt.title('Tranformed signal')
ax2.tick_params(axis='both', which='major', labelsize='x-large')
plt.plot(f, amps, label="transformed")

plt.legend(prop={'size': 'x-large'})
plt.grid()
plt.tight_layout()
plt.show()

```

## Numerical integration

SciPy has a numerical integration package, `scipy.integrate`, which has no equivalent in NumPy. The `quad()` function can integrate a one-variable function between two points. These points can be at infinity. The function uses the simplest numerical integration method: the trapezoid rule.

### Time for action – calculating the Gaussian integral

The **Gaussian integral** is related to the `error()` function (also known in mathematics as `erf`), but has no finite limits. It evaluates to the square root of `pi`.

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Let's calculate the integral with the `quad()` function (for the imports check the file in the code bundle):

```

print("Gaussian integral", np.sqrt(np.pi),
      integrate.quad(lambda x: np.exp(-x**2),
                    -np.inf, np.inf))

```

The return value is the outcome and its error would be as follows:

```

Gaussian integral 1.77245385091 (1.7724538509055159, 1.4202636780944923e-
08)

```

## What just happened?

We calculated the Gaussian integral with the `quad()` function.

## Have a go hero – experiment a bit more

Try out other integration functions from the same package. It should just be a matter of replacing one function call. We should get the same outcome, so you may also want to read the documentation to learn more.

## Interpolation

Interpolation fills in the blanks between known data points in a dataset. The `scipy.interpolate()` function interpolates a function based on experimental data. The `interp1d` class can create a linear or cubic interpolation function. By default, a linear interpolation function is constructed, but if the `kind` parameter is set, a cubic interpolation function is created instead. The `interp2d` class works the same way, but in 2D.

## Time for action – interpolating in one dimension

We will create data points using a `sinc()` function and add some random noise to it. After this, we will do a linear and cubic interpolation and plot the results.

1. Create the data points and add noise to it:

```
x = np.linspace(-18, 18, 36)
noise = 0.1 * np.random.random(len(x))
signal = np.sinc(x) + noise
```
2. Create a linear interpolation function and apply it to an input array with five times as many data points:

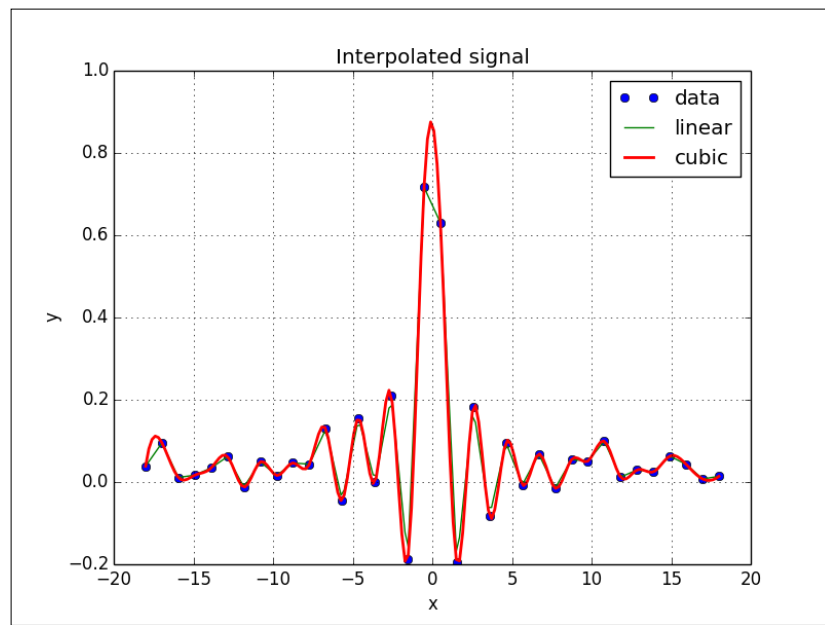
```
interpreted = interpolate.interp1d(x, signal)
x2 = np.linspace(-18, 18, 180)
y = interpreted(x2)
```
3. Do the same as in the previous step, but with cubic interpolation:

```
cubic = interpolate.interp1d(x, signal, kind="cubic")
y2 = cubic(x2)
```
4. Plot the results with `matplotlib`:

```
plt.plot(x, signal, 'o', label="data")
plt.plot(x2, y, '-', label="linear")
```

```
plt.plot(x2, y2, '-', lw=2, label="cubic")
plt.legend()
plt.show()
```

The following diagram is a plot of the data, linear, and cubic interpolation:



### ***What just happened?***

We created a dataset from the `sinc()` function and added noise to it. We then did linear and cubic interpolation using the `interp1d` class of the `scipy.interpolate` module (see `sincinterp.py`):

```
import numpy as np
from scipy import interpolate
import matplotlib.pyplot as plt

x = np.linspace(-18, 18, 36)
noise = 0.1 * np.random.random(len(x))
signal = np.sinc(x) + noise

interpreted = interpolate.interp1d(x, signal)
x2 = np.linspace(-18, 18, 180)
y = interpreted(x2)
```

```
cubic = interpolate.interp1d(x, signal, kind="cubic")
y2 = cubic(x2)

plt.plot(x, signal, 'o', label="data")
plt.plot(x2, y, '-', label="linear")
plt.plot(x2, y2, '-', lw=2, label="cubic")

plt.title('Interpolated signal')
plt.xlabel('x')
plt.ylabel('y')
plt.grid()
plt.legend(loc='best')
plt.show()
```

## Image processing

With SciPy, we can do image processing using the `scipy.ndimage` package. The module contains various image filters and utilities.

### Time for action – manipulating Lena

The `scipy.misc` module is a utility that loads the image of "Lena". This is the image of **Lena Soderberg**, traditionally used for image processing examples. We will apply some filters to this image and rotate it. Perform the following steps to do so:

1. Load the Lena image and display it in a subplot with grayscale colormap:

```
image = misc.lena().astype(np.float32)
plt.subplot(221)
plt.title("Original Image")
img = plt.imshow(image, cmap=plt.cm.gray)
```

Note that we are dealing with a `float32` array.

2. The **median filter** scans the image and replaces each item by the median of neighboring data points. Apply a median filter to the image and display it in a second subplot:

```
plt.subplot(222)
plt.title("Median Filter")
filtered = ndimage.median_filter(image, size=(42,42))
plt.imshow(filtered, cmap=plt.cm.gray)
```

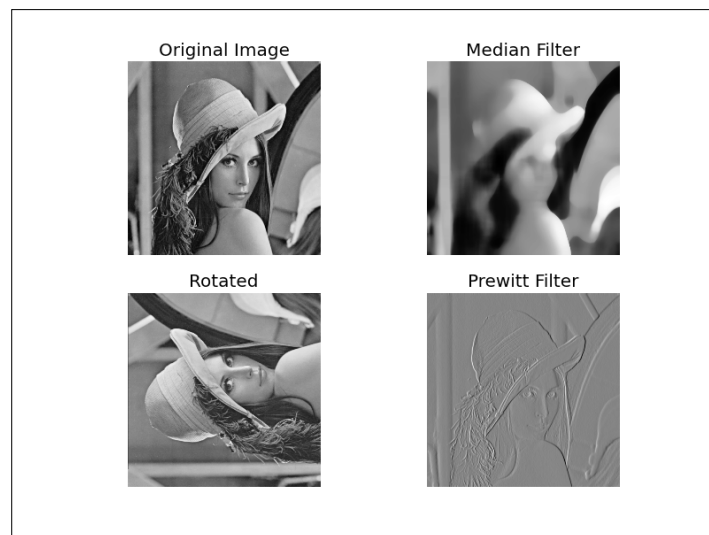
3. Rotate the image and display it in the third subplot:

```
plt.subplot(223)
plt.title("Rotated")
rotated = ndimage.rotate(image, 90)
plt.imshow(rotated, cmap=plt.cm.gray)
```

4. The **Prewitt filter** is based on computing the gradient of image intensity. Apply a Prewitt filter to the image and display it in the fourth subplot:

```
plt.subplot(224)
plt.title("Prewitt Filter")
filtered = ndimage.prewitt(image)
plt.imshow(filtered, cmap=plt.cm.gray)
plt.show()
```

The following are the resulting images:



### ***What just happened?***

We manipulated the image of Lena in several ways using the `scipy.ndimage` module (see `images.py`):

```
from scipy import misc
import numpy as np
import matplotlib.pyplot as plt
from scipy import ndimage
```

```
image = misc.lena().astype(np.float32)

plt.subplot(221)
plt.title("Original Image")
img = plt.imshow(image, cmap=plt.cm.gray)
plt.axis("off")

plt.subplot(222)
plt.title("Median Filter")
filtered = ndimage.median_filter(image, size=(42,42))
plt.imshow(filtered, cmap=plt.cm.gray)
plt.axis("off")

plt.subplot(223)
plt.title("Rotated")
rotated = ndimage.rotate(image, 90)
plt.imshow(rotated, cmap=plt.cm.gray)
plt.axis("off")

plt.subplot(224)
plt.title("Prewitt Filter")
filtered = ndimage.prewitt(image)
plt.imshow(filtered, cmap=plt.cm.gray)
plt.axis("off")
plt.show()
```

## Audio processing

Now that we have done some image processing, you will probably not be surprised that we can do exciting things with WAV files too. Let's download a WAV file and replay it a couple of times. We will skip the explanation of the download part, which is just regular Python.

### Time for action – replaying audio clips

We will download a WAV file of Austin Powers exclaiming "*Smashing baby*". This file can be converted to a NumPy array with the `read()` function from the `scipy.io.wavfile` module. The `write()` function from the same package will be used to create a new WAV file at the end of this section. We will further use the `tile()` function to replay the audio clip several times.

1. Read the file with the `read()` function:  

```
sample_rate, data = wavfile.read(WAV_FILE)
```

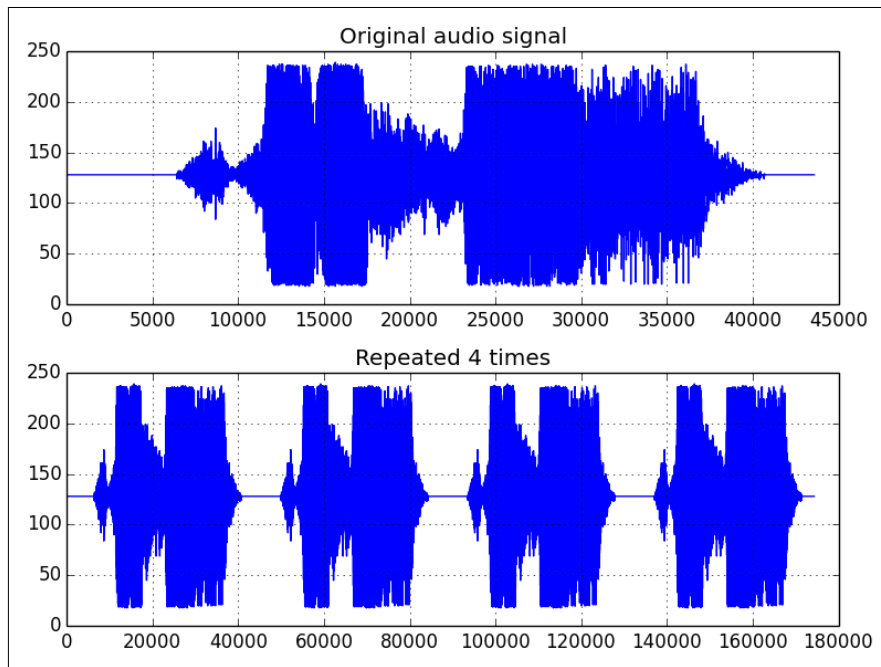
This gives us two items – sample rate and audio data. For this section we are only interested in the audio data.

2. Apply the `tile()` function:
 

```
repeated = np.tile(data, 4)
```
3. Write a new file with the `write()` function:
 

```
wavfile.write("repeated_yababy.wav",
              sample_rate, repeated)
```

The original audio data and the audio clip repeated four times appear in the following plot:



### ***What just happened?***

We read an audio clip, repeated it four times, and then created a new WAV file with the new array (see `repeat_audio.py`):

```
from __future__ import print_function
from scipy.io import wavfile
import matplotlib.pyplot as plt
import urllib.request
import numpy as np

response = urllib.request.urlopen('http://www.thesoundarchive.com/
austinpowers/smashingbaby.wav')
```



```
print(response.info())
WAV_FILE = 'smashingbaby.wav'
filehandle = open(WAV_FILE, 'wb')
filehandle.write(response.read())
filehandle.close()
sample_rate, data = wavfile.read(WAV_FILE)
print("Data type", data.dtype, "Shape", data.shape)

plt.subplot(2, 1, 1)
plt.title("Original audio signal")
plt.plot(data)
plt.grid()

plt.subplot(2, 1, 2)

# Repeat the audio fragment
repeated = np.tile(data, 4)

# Plot the audio data
plt.title("Repeated 4 times")
plt.plot(repeated)
wavfile.write("repeated_yababy.wav",
              sample_rate, repeated)
plt.grid()
plt.tight_layout()
plt.show()
```

## Summary

In this chapter, we only scratched the surface of what is possible with SciPy and SciKits. Still, we learned a bit about file I/O, statistics, signal processing, optimization, interpolation, audio, and image processing.

In the next chapter, we will create some simple, yet fun, games with Pygame—the open source Python game library. In the process, we will learn about NumPy integration with **Pygame**, a machine learning Scikits module, and more.

# 11

## Playing with Pygame

*This chapter is for developers who want to create games quickly and easily with NumPy and Pygame. Basic game development experience would help, but it isn't necessary.*

The things you will learn are as follows:

- ◆ Pygame basics
- ◆ matplotlib integration
- ◆ Surface pixel arrays
- ◆ Artificial intelligence
- ◆ Animation
- ◆ OpenGL

### Pygame

**Pygame** is a Python framework, originally written by **Pete Shinnars**, which, as its name suggests, can be used to create video games. Pygame is free, open source since 2004 and licensed under the GPL license, which means that you are allowed to basically make any type of game. Pygame is built on top of the **Simple DirectMedia Layer (SDL)**. SDL is a C framework that gives access to graphics, sound, keyboard, and other input devices on various operating systems including Linux, Mac OS X, and Windows.

## Time for action – installing Pygame

We will install Pygame in this section. Pygame should be compatible with all Python versions. At the time of writing, there were some incompatibility issues with Python 3, but in all probability, these will be fixed soon.

- ◆ **Installing on Debian and Ubuntu:** Pygame can be found in the Debian archives at <https://packages.qa.debian.org/p/pygame.html>.
- ◆ **Installing on Windows:** From the Pygame website (<http://www.pygame.org/download.shtml>), download the appropriate binary installer for the Python version you are using.
- ◆ **Installing Pygame on the Mac:** Binary Pygame packages for Mac OS X 10.3 and up can be found at <http://www.pygame.org/download.shtml>.
- ◆ **Installing from source:** Pygame is using the `distutils` system for compiling and installing. To start installing Pygame with the default options, simply run the following command:

```
$ python setup.py
```

If you need more information about the available options, type the following:

```
$ python setup.py help
```

- ◆ To compile the code, you need a compiler for your operating system. Setting this up is beyond the scope of this book. More information about compiling Pygame on Windows can be found at <http://pygame.org/wiki/CompileWindows>. For more information about compiling Pygame on Mac OS X, refer to <http://pygame.org/wiki/MacCompile>.

## Hello World

We will create a simple game that we will improve on further in this chapter. As is traditional in programming books, we start with a Hello World! example.

## Time for action – creating a simple game

It's important to notice the so-called main game loop, where all the action happens, and the usage of the `Font` module to render text. In this program, we will manipulate a Pygame `Surface` object that is used for drawing, and we will handle a quit event.

1. First, import the required Pygame modules. If Pygame is installed properly, we should get no errors, otherwise please return to the installation *Time for action*:

```
import pygame, sys
from pygame.locals import *
```

2. Initialize Pygame, create a display of 400 by 300 pixels, and set the window title to Hello world!:

```
pygame.init()
screen = pygame.display.set_mode((400, 300))

pygame.display.set_caption('Hello World!')
```

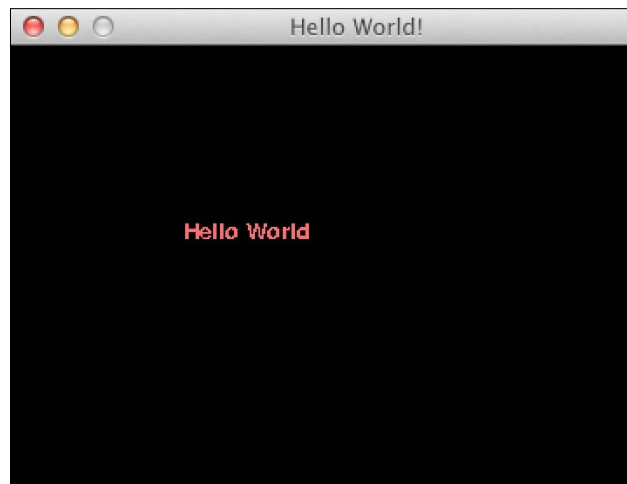
3. Games usually have a game loop, which runs forever until, for instance, a quit event occurs. In this example, only set a label with the text Hello world! at coordinates (100, 100). The text has font size 19 and a red color:

```
while True:
    sysFont = pygame.font.SysFont("None", 19)
    rendered = sysFont.render('Hello World', 0, (255, 100, 100))
    screen.blit(rendered, (100, 100))

    for event in pygame.event.get():
        if event.type == QUIT:
            pygame.quit()
            sys.exit()

    pygame.display.update()
```

We get the following screenshot as an end result:



Following is the complete code for the Hello World! example:

```
import pygame, sys
from pygame.locals import *

pygame.init()
screen = pygame.display.set_mode((400, 300))

pygame.display.set_caption('Hello World!')

while True:
    sysFont = pygame.font.SysFont("None", 19)
    rendered = sysFont.render('Hello World', 0, (255, 100, 100))
    screen.blit(rendered, (100, 100))

    for event in pygame.event.get():
        if event.type == QUIT:
            pygame.quit()
            sys.exit()

    pygame.display.update()
```

### ***What just happened?***

It may not seem like much, but we learned a lot in this section. The functions that passed the review are summarized in the following table:

Function	Description
<code>pygame.init()</code>	This function performs initialization and you must call it before calling other Pygame functions.
<code>pygame.display.set_mode((400, 300))</code>	This function creates a so-called Surface object to draw on. We give this function a tuple representing the dimensions of the surface.
<code>pygame.display.set_caption('Hello World!')</code>	This function sets the window title to a specified string value.
<code>pygame.font.SysFont("None", 19)</code>	This function creates a system font from a comma-separated list of fonts (in this case none) and an integer font size parameter.
<code>sysFont.render('Hello World', 0, (255, 100, 100))</code>	This function draws text on a Surface. The last parameter is a tuple representing the RGB values of a color.
<code>screen.blit(rendered, (100, 100))</code>	This function draws on a Surface.

Function	Description
<code>pygame.event.get()</code>	This function gets a list of <code>Event</code> objects. Events represent a special occurrence in the system, such as a user quitting the game.
<code>pygame.quit()</code>	This function cleans up the resources used by Pygame. Call this function before exiting the game.
<code>pygame.display.update()</code>	This function refreshes the surface.

## Animation

Most games, even the most static ones, have some level of animation. From a programmer's standpoint, animation is nothing more than displaying an object at a different place at a different time, thus simulating movement.

Pygame offers a `Clock` object, which manages how many frames are drawn per second. This ensures that the animation is independent of how fast the user's CPU is.

### Time for action – animating objects with NumPy and Pygame

We will load an image and use NumPy again to define a clockwise path around the screen.

1. Create a Pygame clock as follows:  

```
clock = pygame.time.Clock()
```
2. As part of the source code accompanying this book, there should be a picture of a head. Load this image and move it around on the screen:  

```
img = pygame.image.load('head.jpg')
```
3. Define some arrays to hold the coordinates of the positions, where we would like to put the image during the animation. Since we will move the object, there are four logical sections of the path: `right`, `down`, `left`, and `up`. Each of these sections will have 40 equidistant steps. Initialize all the values in the sections to 0:

```
steps = np.linspace(20, 360, 40).astype(int)
right = np.zeros((2, len(steps)))
down = np.zeros((2, len(steps)))
left = np.zeros((2, len(steps)))
up = np.zeros((2, len(steps)))
```

- 4.** It's straight-forward to set the coordinates of the positions of the image. However, there is one tricky bit to notice—the `[::-1]` notation leads to reversing the order of the array elements:

```
right[0] = steps
right[1] = 20

down[0] = 360
down[1] = steps

left[0] = steps[::-1]
left[1] = 360

up[0] = 20
up[1] = steps[::-1]
```

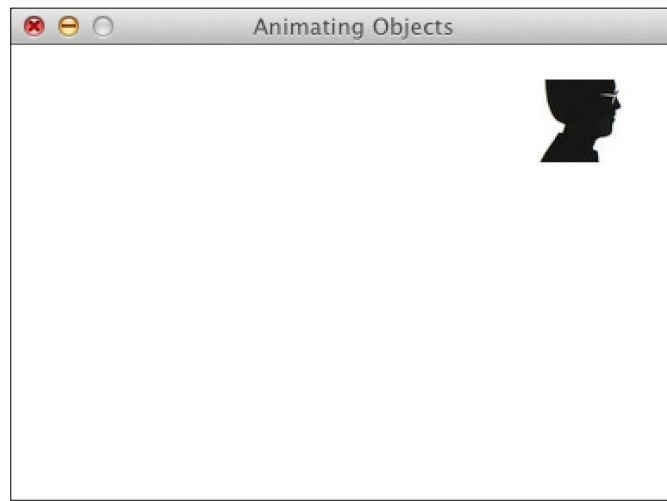
- 5.** We can join the path sections, but before doing this, transpose the arrays with the `T` operator because they are not aligned properly for concatenation:

```
pos = np.concatenate((right.T, down.T, left.T, up.T))
```

- 6.** In the main event loop, let the clock tick at a rate of 30 frames per second:

```
clock.tick(30)
```

A screenshot of the moving head is as follows:



You should be able to watch a movie of this animation at <https://www.youtube.com/watch?v=m2TagGiq1fs>, and it is also part of the code bundle (`animation.mp4`).

The code of this example uses almost everything we have learnt so far, but should still be simple enough to understand:

```
import pygame, sys
from pygame.locals import *
import numpy as np

pygame.init()
clock = pygame.time.Clock()
screen = pygame.display.set_mode((400, 400))

pygame.display.set_caption('Animating Objects')
img = pygame.image.load('head.jpg')

steps = np.linspace(20, 360, 40).astype(int)
right = np.zeros((2, len(steps)))
down = np.zeros((2, len(steps)))
left = np.zeros((2, len(steps)))
up = np.zeros((2, len(steps)))

right[0] = steps
right[1] = 20

down[0] = 360
down[1] = steps

left[0] = steps[::-1]
left[1] = 360

up[0] = 20
up[1] = steps[::-1]

pos = np.concatenate((right.T, down.T, left.T, up.T))
i = 0

while True:
    # Erase screen
    screen.fill((255, 255, 255))

    if i >= len(pos):
        i = 0

    screen.blit(img, pos[i])
    i += 1

    for event in pygame.event.get():
        if event.type == QUIT:
```



```
pygame.quit()
sys.exit()

pygame.display.update()
clock.tick(30)
```

## What just happened?

We learned a bit about animation in this section. The most important concept we learned about is the clock. The following table describes the new functions we used:

Function	Description
<code>pygame.time.Clock()</code>	This creates a game clock.
<code>clock.tick(30)</code>	This function executes a tick of the game clock. Here, 30 is the number of frames per second.

## matplotlib

`matplotlib` is an open source library for easy plotting, which we learned about in *Chapter 9, Plotting with matplotlib*. We can integrate `matplotlib` into a Pygame game and create various plots.

### Time for Action – using matplotlib in Pygame

In this recipe, we take the position coordinates of the previous section and make a graph of them.

1. To integrate `matplotlib` with Pygame, we need to use a non-interactive backend; otherwise `matplotlib` will present us with a GUI window by default. We will import the main `matplotlib` module and call the `use()` function. Call this function immediately after importing the main `matplotlib` module and before importing other `matplotlib` modules:

```
import matplotlib as mpl

mpl.use("Agg")
```

2. We can draw non-interactive plots on a `matplotlib` canvas. Creating this canvas requires imports, creating a figure and a subplot. Specify the figure to be 3 by 3 inches large. More details can be found at the end of this recipe:

```
import matplotlib.pyplot as plt
import matplotlib.backends.backend_agg as agg
```

```
fig = plt.figure(figsize=[3, 3])
ax = fig.add_subplot(111)
canvas = agg.FigureCanvasAgg(fig)
```

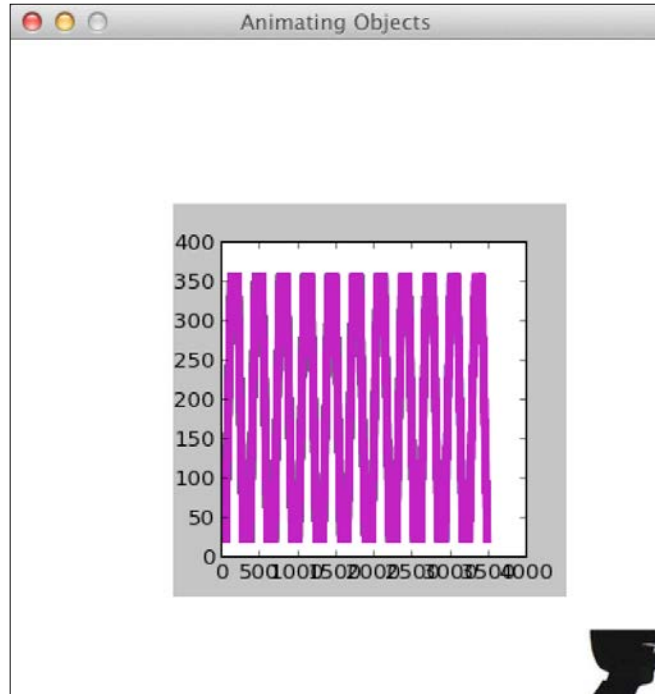
- 3.** In non-interactive mode, plotting data is a bit more complicated than in the default mode. Since we need to plot repeatedly, it makes sense to organize the plotting code in a function. Pygame eventually draws the plot on the canvas. The canvas adds a bit of complexity to our setup. At the end of this example, you can find more detailed explanation of the functions:

```
def plot(data):
    ax.plot(data)
    canvas.draw()
    renderer = canvas.get_renderer()

    raw_data = renderer.tostring_rgb()
    size = canvas.get_width_height()

    return pygame.image.fromstring(raw_data, size, "RGB")
```

The following screenshot shows the animation in action. You can also view a screencast in the code bundle (`matplotlib.mp4`) and on YouTube at: <https://www.youtube.com/watch?v=t6qTeXtnl4>.



We get the following code after the changes:

```
import pygame, sys
from pygame.locals import *
import numpy as np
import matplotlib as mpl

mpl.use("Agg")

import matplotlib.pyplot as plt
import matplotlib.backends.backend_agg as agg

fig = plt.figure(figsize=[3, 3])
ax = fig.add_subplot(111)
canvas = agg.FigureCanvasAgg(fig)

def plot(data):
    ax.plot(data)
    canvas.draw()
    renderer = canvas.get_renderer()

    raw_data = renderer.tostring_rgb()
    size = canvas.get_width_height()

    return pygame.image.fromstring(raw_data, size, "RGB")

pygame.init()
clock = pygame.time.Clock()
screen = pygame.display.set_mode((400, 400))

pygame.display.set_caption('Animating Objects')
img = pygame.image.load('head.jpg')

steps = np.linspace(20, 360, 40).astype(int)
right = np.zeros((2, len(steps)))
down = np.zeros((2, len(steps)))
left = np.zeros((2, len(steps)))
up = np.zeros((2, len(steps)))

right[0] = steps
right[1] = 20

down[0] = 360
down[1] = steps
```

```

left[0] = steps[::-1]
left[1] = 360

up[0] = 20
up[1] = steps[::-1]

pos = np.concatenate((right.T, down.T, left.T, up.T))
i = 0
history = np.array([])
surf = plot(history)

while True:
    # Erase screen
    screen.fill((255, 255, 255))

    if i >= len(pos):
        i = 0
        surf = plot(history)

    screen.blit(img, pos[i])
    history = np.append(history, pos[i])
    screen.blit(surf, (100, 100))

    i += 1

    for event in pygame.event.get():
        if event.type == QUIT:
            pygame.quit()
            sys.exit()

    pygame.display.update()
    clock.tick(30)

```

### ***What just happened?***

The following table explains the plotting related functions:

Function	Description
<code>mpl.use("Agg")</code>	This function specifies to use the non-interactive backend
<code>plt.figure(figsize=[3, 3])</code>	This function creates a figure of 3 by 3 inches
<code>agg.FigureCanvasAgg(fig)</code>	This function creates a canvas in non-interactive mode
<code>canvas.draw()</code>	This function draws on the canvas
<code>canvas.get_renderer()</code>	This function gets a renderer for the canvas

## Surface pixels

The Pygame `surfarray` module handles the conversion between Pygame `Surface` objects and NumPy arrays. As you may recall, NumPy can manipulate big arrays in a fast and efficient manner.

### Time for Action – accessing surface pixel data with NumPy

In this section, we will tile a small image to fill the game screen.

1. The `array2d()` function copies pixels into a two-dimensional array (and there is a similar function for three-dimensional arrays). Copy the pixels from the avatar image into an array:

```
pixels = pygame.surfarray.array2d(img)
```

2. Create the game screen from the shape of the pixels array using the `shape` attribute of the array. Make the screen seven times larger in both directions:

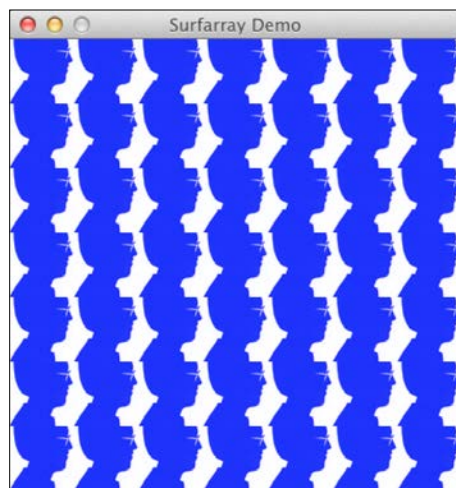
```
X = pixels.shape[0] * 7
Y = pixels.shape[1] * 7
screen = pygame.display.set_mode((X, Y))
```

3. Tiling the image is easy with the NumPy `tile()` function. The data needs to be converted into integer values, because Pygame defines colors as integers:

```
new_pixels = np.tile(pixels, (7, 7)).astype(int)
```

4. The `surfarray` module has a special function `blit_array()` to display the array on the screen:

```
pygame.surfarray.blit_array(screen, new_pixels)
```



The following code performs the tiling of the image:

```
import pygame, sys
from pygame.locals import *
import numpy as np

pygame.init()
img = pygame.image.load('head.jpg')
pixels = pygame.surfarray.array2d(img)
X = pixels.shape[0] * 7
Y = pixels.shape[1] * 7
screen = pygame.display.set_mode((X, Y))
pygame.display.set_caption('Surfarray Demo')
new_pixels = np.tile(pixels, (7, 7)).astype(int)

while True:
    screen.fill((255, 255, 255))
    pygame.surfarray.blit_array(screen, new_pixels)

    for event in pygame.event.get():
        if event.type == QUIT:
            pygame.quit()
            sys.exit()

    pygame.display.update()
```

### ***What just happened?***

Following is a brief description of the new functions and attributes we used:

Function	Description
<code>pygame.surfarray.array2d(img)</code>	This function copies pixel data into a two-dimensional array
<code>pygame.surfarray.blit_array(screen, new_pixels)</code>	This function displays array values on the screen

## Artificial Intelligence

Often we need to mimic intelligent behavior within a game. The `scikit-learn` project aims to provide an API for machine learning, and what I like most about it is its amazing documentation. We can install `scikit-learn` with the package manager of our operating system, though this option may or may not be available, depending on your operating system, but should be the most convenient route. Windows users can just download an installer from the project website. On Debian and Ubuntu, the project is called `python-sklearn`. On MacPorts, the ports are called `py26-scikits-learn` and `py27-scikits-learn`. We can also install from source or using `easy_install`. There are third-party distributions from **Python(x,y)**, **Enthought**, and **NetBSD**.

We can install `scikit-learn` by typing at command line:

```
$ [sudo] pip install -U scikit-learn
```

We can also type the following instead of the preceding line:

```
$ [sudo] easy_install -U scikit-learn
```

This may not work because of permissions, so you might need to put `sudo` in front of the commands or log in as admin.

### Time for Action – clustering points

We will generate some random points and cluster them, which means that the points that are close to each other are put into the same cluster. This is just one of the many techniques that you can apply with `scikit-learn`. **Clustering** is a type of machine learning algorithm, which aims to group items based on similarities. Next, we will calculate a square affinity matrix. An **affinity matrix** is a matrix containing affinity values: for instance, the distances between points. Finally, we will cluster the points with the `AffinityPropagation` class from `scikit-learn`.

1. Generate 30 random point positions within a square of 400 by 400 pixels:
2. Calculate the affinity matrix using the Euclidean distance to the origin as the affinity metric:

```
positions = np.random.randint(0, 400, size=(30, 2))  
  
positions_norms = np.sum(positions ** 2, axis=1)  
S = - positions_norms[:, np.newaxis] -  
    positions_norms[np.newaxis, :] + 2 *  
    np.dot(positions, positions.T)
```

3. Give the `AffinityPropagation` class the result from the previous step. This class labels the points with the appropriate cluster number:

```
aff_pro = sklearn.cluster.AffinityPropagation().fit(S)
labels = aff_pro.labels_
```

4. Draw polygons for each cluster. The function involved requires a list of points, a color (let's paint it red), and a surface:

```
pygame.draw.polygon(screen, (255, 0, 0), polygon_points[i])
```

The result is a bunch of polygons for each cluster, as shown in the following picture:



The clustering example code is as follows:

```
import numpy as np
import sklearn.cluster
import pygame, sys
from pygame.locals import *

np.random.seed(42)
positions = np.random.randint(0, 400, size=(30, 2))

positions_norms = np.sum(positions ** 2, axis=1)
S = - positions_norms[:, np.newaxis] - positions_norms[np.newaxis, :] + 2 * np.dot(positions, positions.T)
```



```
aff_pro = sklearn.cluster.AffinityPropagation().fit(S)
labels = aff_pro.labels_

polygon_points = []

for i in xrange(max(labels) + 1):
    polygon_points.append([])

# Sorting points by cluster
for label, position in zip(labels, positions):
    polygon_points[labels[i]].append(positions[i])

pygame.init()
screen = pygame.display.set_mode((400, 400))

while True:
    for point in polygon_points:
        pygame.draw.polygon(screen, (255, 0, 0), point)

    for event in pygame.event.get():
        if event.type == QUIT:
            pygame.quit()
            sys.exit()

    pygame.display.update()
```

## ***What just happened?***

The most important lines in the artificial intelligence example are described in more detail in the following table:

Function	Description
<code>sklearn.cluster.AffinityPropagation().fit(S)</code>	This function creates an <code>AffinityPropagation</code> object and performs a fit using an affinity matrix
<code>pygame.draw.polygon(screen, (255, 0, 0), point)</code>	This function draws a polygon given a surface, a color (red in this case), and a list of points

## OpenGL and Pygame

OpenGL specifies an API for two-dimensional and three-dimensional computer graphics. The API consists of functions and constants. We will be concentrating on the Python implementation called `PyOpenGL`. Install `PyOpenGL` with the following command:

```
$ [sudo] pip install PyOpenGL PyOpenGL_accelerate
```

You might need to have root access to execute this command. The corresponding `easy_install` command is as follows:

```
$ [sudo] easy_install PyOpenGL PyOpenGL_accelerate
```

### Time for Action – drawing the Sierpinski gasket

For the purpose of demonstration, we will draw a **Sierpinski gasket**, also known as **Sierpinski triangle** or **Sierpinski Sieve** with **OpenGL**. This is a fractal pattern in the shape of a triangle created by the mathematician **Waclaw Sierpinski**. The triangle is obtained via a recursive and, in principle infinite procedure.

1. First, start out by initializing some of the OpenGL related primitives. This includes setting the display mode and background color. A line-by-line explanation is given at the end of this section:

```
def display_openGL(w, h):
    pygame.display.set_mode((w,h),
        pygame.OPENGL|pygame.DOUBLEBUF)

    glClearColor(0.0, 0.0, 0.0, 1.0)
    glClear(GL_COLOR_BUFFER_BIT|GL_DEPTH_BUFFER_BIT)

    gluOrtho2D(0, w, 0, h)
```

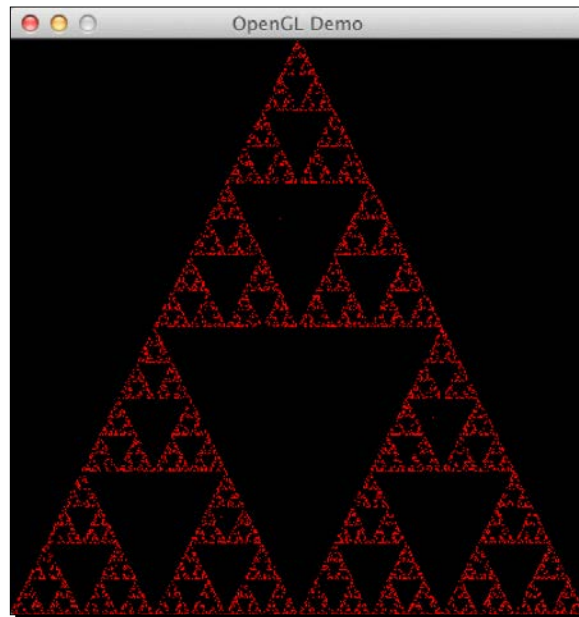
2. The algorithm requires us to display points, the more the better. First, we set the drawing color to red. Second, we define the vertices (I call them points myself) of a triangle. Then, we define random indices, which are to be used to choose one of the three triangle vertices. We pick a random point somewhere in the middle—it doesn't really matter where. After this, draw points halfway between the previous point and one of the vertices picked at random. Finally, flush the result:

```
glColor3f(1.0, 0, 0)
vertices = np.array([[0, 0], [DIM/2, DIM], [DIM, 0]])
NPOINTS = 9000
indices = np.random.random_integers(0, 2, NPOINTS)
point = [175.0, 150.0]
```

```
for i in xrange(NPOINTS):
    glBegin(GL_POINTS)
    point = (point + vertices[indices[i]])/2.0
    glVertex2fv(point)
    glEnd()

glFlush()
```

The Sierpinski triangle looks like the following:



The full Sierpinski gasket demo code with all the imports is as follows:

```
import pygame
from pygame.locals import *
import numpy as np

from OpenGL.GL import *
from OpenGL.GLU import *

def display_openGL(w, h):
    pygame.display.set_mode((w,h), pygame.OPENGL|pygame.DOUBLEBUF)

    glClearColor(0.0, 0.0, 0.0, 1.0)
    glClear(GL_COLOR_BUFFER_BIT|GL_DEPTH_BUFFER_BIT)
```

```

gluOrtho2D(0, w, 0, h)

def main():
    pygame.init()
    pygame.display.set_caption('OpenGL Demo')
    DIM = 400
    display_openGL(DIM, DIM)
    glColor3f(1.0, 0, 0)
    vertices = np.array([[0, 0], [DIM/2, DIM], [DIM, 0]])
    NPOINTS = 9000
    indices = np.random.random_integers(0, 2, NPOINTS)
    point = [175.0, 150.0]

    for i in xrange(NPOINTS):
        glBegin(GL_POINTS)
        point = (point + vertices[indices[i]])/2.0
        glVertex2fv(point)
        glEnd()

    glFlush()
    pygame.display.flip()

    while True:
        for event in pygame.event.get():
            if event.type == QUIT:
                return

if __name__ == '__main__':
    main()

```

### ***What just happened?***

As promised, the following is a line-by-line explanation of the most important parts of the example:

Function	Description
<code>pygame.display.set_mode((w,h), pygame.OPENGL pygame.DOUBLEBUF)</code>	This function sets the display mode to the required width, height, and OpenGL display
<code>glClear(GL_COLOR_BUFFER_BIT GL_DEPTH_BUFFER_BIT)</code>	This function clears the buffers using a mask. Here we clear the color buffer and depth buffer bits
<code>gluOrtho2D(0, w, 0, h)</code>	This function defines a two-dimensional orthographic projection matrix with the coordinates of the left, right, top, and bottom clipping planes

Function	Description
<code>glColor3f(1.0, 0, 0)</code>	This function defines the current drawing color using three float values for RGB (red, green, blue). In this case, we will be painting in red
<code>glBegin(GL_POINTS)</code>	This function delimits the vertices of primitives or a group of primitives. Here the primitives are points
<code>glVertex2fv(point)</code>	This function renders a point given a vertex
<code>glEnd()</code>	This function closes a section of code started with <code>glBegin()</code>
<code>glFlush()</code>	This function forces the execution of GL commands

## Simulation game with Pygame

As a last example, we will simulate life with **Conway's Game of Life**. The original game of life is based on a few basic rules. We start out with a random configuration on a two-dimensional square grid. Each cell in the grid can be either dead or alive. This state depends on the neighbors of the cell. You can read more about the rules at [http://en.wikipedia.org/wiki/Conway%27s\\_Game\\_of\\_Life#Rules](http://en.wikipedia.org/wiki/Conway%27s_Game_of_Life#Rules) At each step in time, the following transitions occur:

1. Live cells with less than two live neighbors die.
2. Live cells with two or three live neighbors live on to the next generation.
3. Live cells with more than three live neighbors die.
4. Dead cells with exactly three live neighbors become a live cell.

Convolution can be used to evaluate the basic rules of the game. We need the SciPy package for the convolution process.

### Time for Action – simulating life

The following code is an implementation of the Game of Life, with some modifications:

- ◆ Clicking once with the mouse draws a cross until we click again
- ◆ The *r* key resets the grid to a random state
- ◆ Pressing *b* creates blocks based on the mouse position
- ◆ *g* creates gliders

The most important data structure in the code is a two-dimensional array, holding the color values of the pixels on the game screen. This array is initialized with random values and then recalculated for each iteration of the game loop. Find more information about the involved functions in the next section.

1. To evaluate the rules, use the convolution as follows:

```
def get_pixar(arr, weights):
    states = ndimage.convolve(arr, weights, mode='wrap')

    bools = (states == 13) | (states == 12 ) | (states == 3)

    return bools.astype(int)
```

2. Draw a cross using the basic indexing tricks that we learned in *Chapter 2, Beginning with NumPy Fundamentals*:

```
def draw_cross(pixar):
    (posx, posy) = pygame.mouse.get_pos()
    pixar[posx, :] = 1
    pixar[:, posy] = 1
```

3. Initialize the grid with random values:

```
def random_init(n):
    return np.random.random_integers(0, 1, (n, n))
```

The following is the code in its entirety:

```
from __future__ import print_function
import os, pygame
from pygame.locals import *
import numpy as np
from scipy import ndimage

def get_pixar(arr, weights):
    states = ndimage.convolve(arr, weights, mode='wrap')

    bools = (states == 13) | (states == 12 ) | (states == 3)

    return bools.astype(int)

def draw_cross(pixar):
    (posx, posy) = pygame.mouse.get_pos()
    pixar[posx, :] = 1
    pixar[:, posy] = 1
```

```
def random_init(n):
    return np.random.random_integers(0, 1, (n, n))

def draw_pattern(pixar, pattern):
    print(pattern)

    if pattern == 'glider':
        coords = [(0,1), (1,2), (2,0), (2,1), (2,2)]
    elif pattern == 'block':
        coords = [(3,3), (3,2), (2,3), (2,2)]
    elif pattern == 'exploder':
        coords = [(0,1), (1,2), (2,0), (2,1), (2,2), (3,3)]
    elif pattern == 'fpentomino':
        coords = [(2,3), (3,2), (4,2), (3,3), (3,4)]

    pos = pygame.mouse.get_pos()

    xs = np.arange(0, pos[0], 10)
    ys = np.arange(0, pos[1], 10)

    for x in xs:
        for y in ys:
            for i, j in coords:
                pixar[x + i, y + j] = 1

def main():
    pygame.init ()
    N = 400
    pygame.display.set_mode((N, N))
    pygame.display.set_caption("Life Demo")

    screen = pygame.display.get_surface()

    pixar = random_init(N)
    weights = np.array([[1,1,1], [1,10,1], [1,1,1]])

    cross_on = False

    while True:
        pixar = get_pixar(pixar, weights)

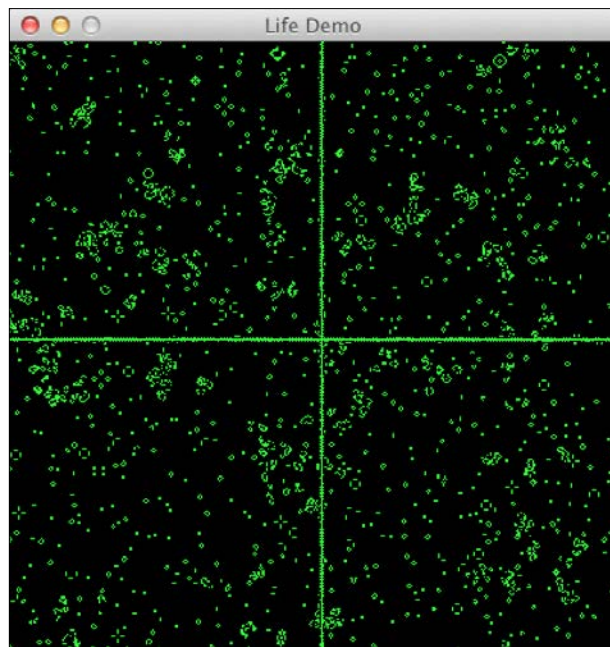
        if cross_on:
            draw_cross(pixar)

        pygame.surfarray.blit_array(screen, pixar * 255 ** 3)
        pygame.display.flip()
```

```
for event in pygame.event.get():
    if event.type == QUIT:
        return
    if event.type == MOUSEBUTTONDOWN:
        cross_on = not cross_on
    if event.type == KEYDOWN:
        if event.key == ord('r'):
            pixar = random_init(N)
            print("Random init")
        if event.key == ord('g'):
            draw_pattern(pixar, 'glider')
        if event.key == ord('b'):
            draw_pattern(pixar, 'block')
        if event.key == ord('e'):
            draw_pattern(pixar, 'exploder')
        if event.key == ord('f'):
            draw_pattern(pixar, 'fpentomino')

if __name__ == '__main__':
    main()
```

You should be able to view a screencast from the code bundle ([life.mp4](#)) or on YouTube (<https://www.youtube.com/watch?v=NNsU-yWTkXM>). A screenshot of the game in action is as follows:





## **What just happened?**

We used some NumPy and SciPy functions that need explaining:

Function	Description
<code>ndimage.convolve(arr, weights, mode='wrap')</code>	This function applies the convolve operation on the given array, using weights in the wrap mode. The mode has to do with the array borders.
<code>bools.astype(int)</code>	This function converts the array of Booleans to integers.
<code>np.arange(0, pos[0], 10)</code>	This function creates an array from 0 to <code>pos[0]</code> in steps of 10. So, if <code>pos[0]</code> is equal to 1000, we will get 0, 10, 20, ... 990.

## **Summary**

You might find the mention of Pygame in this book a bit odd. However, after reading this chapter, I hope you realized that NumPy and Pygame go well together. Games after all involve lots of computation for which NumPy and SciPy are ideal choices, and they also require artificial intelligence capabilities as found in `scikit-learn`. In any event, making games is fun and we hope this last chapter was the equivalent of a nice dessert or coffee after a ten-course meal! If you are still hungry for more, please check out *NumPy Cookbook, Second Edition*, Ivan Idris, Packt Publishing, which builds further on this book with minimum overlap.

# A

## Pop Quiz Answers

### Chapter 1, NumPy Quick Start

#### Pop quiz – functioning of the arange() function

What does <code>arange(5)</code> do?	It creates a NumPy array with values 0-4 The created NumPy array has values 0, 1, 2, 3, and 4
--------------------------------------	--

### Chapter 2, Beginning with NumPy Fundamentals

#### Pop quiz – the shape of ndarray

How is the shape of an ndarray stored?	It is stored in a tuple
--	-------------------------

## Chapter 3, Getting Familiar with Commonly Used Functions

### Pop quiz – computing the weighted average

Which function returns the weighted average of an array?	average
--	---------

## Chapter 4, Convenience Functions for Your Convenience

### Pop quiz – calculating covariance

Which function returns the covariance of two arrays?	cov
--	-----

## Chapter 5, Working with Matrices and ufuncs

### Pop quiz – defining a matrix with a string

What is the row delimiter in a string accepted by the <code>mat</code> and <code>bmat</code> functions?	Semicolon
---	-----------

## Chapter 6, Move Further with NumPy Modules

### Pop quiz – creating a matrix

Which function can create matrices?	mat
-------------------------------------	-----

---

## Chapter 7, Peeking into Special Routines

### Pop quiz – generating random numbers

Which NumPy module deals with random numbers?	random
---	--------

## Chapter 8, Assuring Quality with Testing

### Pop quiz – specifying decimal precision

Which parameter of the <code>assert_almost_equal</code> function specifies the decimal precision?	decimal
---	---------

## Chapter 9, Plotting with matplotlib

### Pop quiz – the `plot()` function

What does the <code>plot</code> function do?	It does neither 1, 2, or 3
--	----------------------------

## Chapter 10, When NumPy Is Not Enough –SciPy and Beyond

### Pop quiz – loading `.mat` files

Which function loads <code>.mat</code> files?	loadmat
---	---------



# B

## Additional Online Resources

This appendix contains links to the relevant websites.

### Python

- ◆ Learn Python the Hard Way (for Python 2) at <http://learnpythonthehardway.org/>
- ◆ Dive Into Python 3 (for Python 3) at <http://www.diveintopython3.net/>
- ◆ Beginner's Guide to Python at <https://wiki.python.org/moin/BeginnersGuide>
- ◆ Non-programmers Tutorial for Python 3 can be found at [http://en.wikibooks.org/wiki/Non-Programmer%27s\\_Tutorial\\_for\\_Python\\_3](http://en.wikibooks.org/wiki/Non-Programmer%27s_Tutorial_for_Python_3)
- ◆ A Byte of Python is available at <http://www.swaroopch.com/notes/python/>
- ◆ An Introduction to Interactive Programming in Python can be found at <https://www.coursera.org/course/interactivepython1>
- ◆ Learn Python online by Code Mentor at <https://www.codementor.io/learn-python-online>
- ◆ Learn Python by visualizing code execution at <http://pythontutor.com/>
- ◆ Find Codecademy Python exercises at <http://www.codecademy.com/tracks/python>
- ◆ Google's Python class is available at <https://developers.google.com/edu/python/>
- ◆ A Python style guide from Google can be found at <https://google-styleguide.googlecode.com/svn/trunk/pyguide.html>
- ◆ The IPython website can be found at <http://ipython.org/>
- ◆ matplotlib a Python plotting library at <http://matplotlib.org/>

- ◆ NumPy and SciPy documentation can be accessed at <http://docs.scipy.org/doc/>
- ◆ NumPy and SciPy mailing lists can be found at <http://www.scipy.org/scipylib/mailling-lists.html>

## **Mathematics and statistics**

- ◆ Linear algebra tutorials are available from Khan Academy at <https://www.khanacademy.org/math/linear-algebra>
- ◆ Pre-calculus tutorials from Khan Academy are available at <https://www.khanacademy.org/math/precalculus>
- ◆ Probability and statistics tutorials from Khan Academy can be found at <https://www.khanacademy.org/math/probability>
- ◆ Trigonometry tutorials from Khan Academy can be found at <https://www.khanacademy.org/math/trigonometry>
- ◆ Access Alcumus by **Art of Problem Solving(AoPS)** at <http://www.artofproblemsolving.com/alcumus>
- ◆ Find the Pre-Calculus Coursera course at <https://www.coursera.org/course/precalculus>
- ◆ The Coursera course on linear algebra, which uses Python, can be found at <https://www.coursera.org/course/matrix>
- ◆ An introduction to probability by **Harvard University** can be accessed at <https://itunes.apple.com/us/course/statistics-110-probability/id502492375>
- ◆ The statistics wikibook is available at <https://en.wikibooks.org/wiki/Statistics>
- ◆ The Electronic Statistics Textbook. Tulsa, OK: StatSoft. WEB can be found at: <http://www.statsoft.com/Textbook>

# C

## NumPy Functions' References

This appendix contains a list of useful NumPy functions and their descriptions.

- ◆ `numpy.apply_along_axis(func1d, axis, arr, *args)`: Applies the function `func1d` along an axis on 1D slices of `arr`.
- ◆ `numpy.arange([start,] stop[, step,], dtype=None)`: Creates a NumPy array with evenly spaced values within a specified range.
- ◆ `numpy.argsort(a, axis=-1, kind='quicksort', order=None)`: Returns the indices that would sort the input array.
- ◆ `numpy.argmax(a, axis=None)`: Returns the indices of the maximum values along an axis.
- ◆ `numpy.argmin(a, axis=None)`: Returns the indices of the minimum values along an axis.
- ◆ `numpy.argwhere(a)`: Finds the indices of non-zero elements.
- ◆ `numpy.array(object, dtype=None, copy=True, order=None, subok=False, ndmin=0)`: Creates a NumPy array from an array-like sequence, such as a Python list.
- ◆ `numpy.testing.assert_allclose(actual, desired, rtol=1e-07, atol=0, err_msg='', verbose=True)`: Raises an error if two objects are unequal up to a specified precision.
- ◆ `numpy.testing.assert_almost_equal()`: Raises an exception if two numbers are not equal up to a specified precision.
- ◆ `numpy.testing.assert_approx_equal()`: Raises an exception if two numbers are not equal up to a certain significance.
- ◆ `numpy.testing.assert_array_almost_equal()`: Raises an exception if two arrays are not equal up to a specified precision.



- ◆ `numpy.testing.assert_array_almost_equal_nulp(x, y, nulp=1)`: Compares arrays to their **unit of least precision (ULP)**.
- ◆ `numpy.testing.assert_array_equal()`: Raises an exception if two arrays are not equal.
- ◆ `numpy.testing.assert_array_less()`: Raises an exception if two arrays do not have the same shape, and the elements of the first array are strictly less than the elements of the second array.
- ◆ `numpy.testing.assert_array_max_ulp(a, b, maxulp=1, dtype=None)`: Determines whether the array elements differ by, at most, a specified number of ULP.
- ◆ `numpy.testing.assert_equal()`: Tests whether two NumPy arrays are equal.
- ◆ `numpy.testing.assert_raises()`: Fails if a specified exception is not raised by a callable invoked with defined arguments.
- ◆ `numpy.testing.assert_string_equal()`: Asserts that two strings are equal.
- ◆ `numpy.testing.assert_warns()`: Fails if a specified warning is not thrown.
- ◆ `numpy.bartlett(M)`: Returns the Bartlett window with *M* points. This window is similar to a triangular window.
- ◆ `numpy.random.binomial(n, p, size=None)`: Draws random samples from the binomial distribution.
- ◆ `numpy.bitwise_and(x1, x2[, out])`: Calculates the bit-wise AND of arrays.
- ◆ `numpy.bitwise_xor(x1, x2[, out])`: Calculates the bit-wise XOR of arrays.
- ◆ `numpy.blackman(M)`: Returns a Blackman window with *M* points, which is close to optimal and a little bit worse than a Kaiser window.
- ◆ `numpy.column_stack(tup)`: Stacks 1D arrays provided as a tuple column wise.
- ◆ `numpy.concatenate((a1, a2, ...), axis=0)`: Concatenates a sequence of arrays.
- ◆ `numpy.convolve(a, v, mode='full')`: Computes the linear convolution of 1D arrays.
- ◆ `numpy.dot(a, b, out=None)`: Calculates the dot product of two arrays.
- ◆ `numpy.diff(a, n=1, axis=-1)`: Computes the *n*th difference for a given axis.
- ◆ `numpy.dsplit(ary, indices_or_sections)`: Splits an array into subarrays along the third axis.
- ◆ `numpy.dstack(tup)`: Stacks arrays given as a tuple along the third axis.
- ◆ `numpy.eye(N, M=None, k=0, dtype=<type 'float'>)`: Returns the identity matrix.

- ◆ `numpy.extract(condition, arr)`: Selects elements of an array using a condition.
- ◆ `numpy.fft.fftshift(x, axes=None)`: Shifts the zero-frequency component of a signal to the center of the spectrum.
- ◆ `numpy.hamming(M)`: Returns the Hamming window with `M` points.
- ◆ `numpy.hanning(M)`: Returns the Hanning window with `M` points.
- ◆ `numpy.hstack(tup)`: Stacks arrays given as a tuple horizontally.
- ◆ `numpy.isreal(x)`: Returns a Boolean array, where `True` corresponds to an element of the input array, which is a real number (as opposed to a complex number).
- ◆ `numpy.kaiser(M, beta)`: Returns a Kaiser window with `M` points for a given `beta` parameter.
- ◆ `numpy.load(file, mmap_mode=None)`: Loads NumPy arrays or pickled objects from `.npy`, `.npz` or pickles. A memory-mapped array is stored in the filesystem and doesn't have to be completely loaded in memory. This is especially useful for large arrays.
- ◆ `numpy.loadtxt(fname, dtype=<type 'float'>, comments='#', delimiter=None, converters=None, skiprows=0, usecols=None, unpack=False, ndmin=0)`: Loads data from a text file into a NumPy array.
- ◆ `numpy.lexsort(keys, axis=-1)`: Sorts using multiple keys.
- ◆ `numpy.linspace(start, stop, num=50, endpoint=True, retstep=False, dtype=None)`: Returns evenly spaced numbers over an interval.
- ◆ `numpy.max(a, axis=None, out=None, keepdims=False)`: Returns the maximum of an array along an axis.
- ◆ `numpy.mean(a, axis=None, dtype=None, out=None, keepdims=False)`: Calculates the arithmetic mean along the given axis.
- ◆ `numpy.median(a, axis=None, out=None, overwrite_input=False)`: Calculates the median along the given axis.
- ◆ `numpy.meshgrid(*xi, **kwargs)`: Returns coordinate matrices for coordinate vectors. For instance:  
In: `numpy.meshgrid([1, 2], [3, 4])`  
Out:  
`[array([[1, 2],  
 [1, 2]]), array([[3, 3],  
 [4, 4]])]`

- ◆ `numpy.min(a, axis=None, out=None, keepdims=False)`: Returns the minimum of an array along an axis.
- ◆ `numpy.msort(a)`: Returns a copy of an array sorted along the first axis.
- ◆ `numpy.nanargmax(a, axis=None)`: Returns the indices of the maximums given an axis ignoring NaNs.
- ◆ `numpy.nanargmin(a, axis=None)`: Returns the indices of the minimums given an axis ignoring NaNs.
- ◆ `numpy.nonzero(a)`: Returns indices of non-zero array elements.
- ◆ `numpy.ones(shape, dtype=None, order='C')`: Creates a NumPy array of specified shape and data type, containing 1s.
- ◆ `numpy.piecewise(x, condlist, funclist, *args, **kw)`: Evaluates a function piecewise.
- ◆ `numpy.polyder(p, m=1)`: Differentiates a polynomial to a given order.
- ◆ `numpy.polyfit(x, y, deg, rcond=None, full=False, w=None, cov=False)`: Performs a least squares polynomial fit.
- ◆ `numpy.polysub(a1, a2)`: Subtracts polynomials.
- ◆ `numpy.polyval(p, x)`: Evaluates a polynomial at specified values.
- ◆ `numpy.prod(a, axis=None, dtype=None, out=None, keepdims=False)`: Returns the product of array elements over a specified axis.
- ◆ `numpy.ravel(a, order='C')`: Flattens an array or returns a copy if necessary.
- ◆ `numpy.reshape(a, newshape, order='C')`: Changes the shape of a NumPy array.
- ◆ `numpy.row_stack(tup)`: Stacks arrays row wise.
- ◆ `numpy.save(file, arr)`: Saves a NumPy array to a file in the NumPy `.npy` format.
- ◆ `numpy.savetxt(fname, X, fmt='%18e', delimiter=' ', newline='\n', header='', footer='', comments='# ')`: Saves a NumPy array to a text file.
- ◆ `numpy.sinc(a)`: Computes the `sinc` function.
- ◆ `numpy.sort_complex(a)`: Sorts array elements with the real part first, then followed by the imaginary part.
- ◆ `numpy.split(a, indices_or_sections, axis=0)`: Splits an array into subarrays.

- ◆ `numpy.std(a, axis=None, dtype=None, out=None, ddof=0, keepdims=False)`: Returns the standard deviation along the given axis.
- ◆ `numpy.take(a, indices, axis=None, out=None, mode='raise')`: Selects elements from an array using specified indices.
- ◆ `numpy.vsplit(a, indices_or_sections)`: Splits an array into subarrays vertically.
- ◆ `numpy.vstack(tup)`: Stacks arrays vertically.
- ◆ `numpy.where(condition, [x, y])`: Selects array elements from input arrays based on a Boolean condition.
- ◆ `numpy.zeros(shape, dtype=float, order='C')`: Creates a NumPy array of specified shape and data type, containing zeros.



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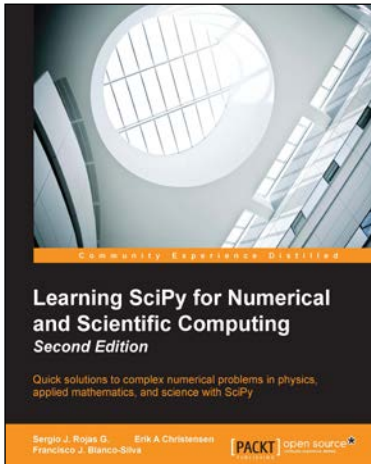
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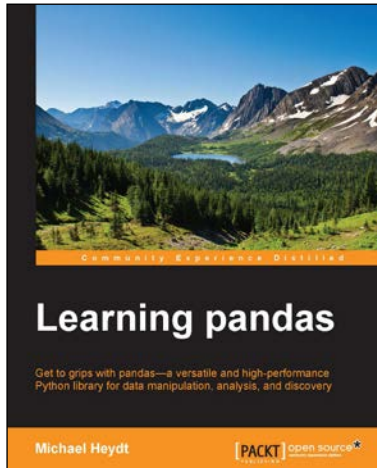
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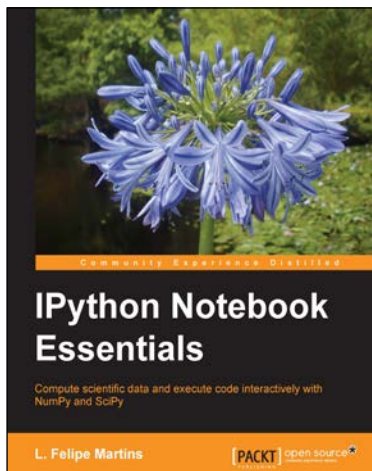


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